Finite Element Method and Application **Sharif University of Technology**
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Assignment Number 5

1) Transient two-dimensional variational principle for a field problem of fluid at any time "t" is given by:

$$J(\phi) = \frac{1}{2}\iint_A (\phi_x^{\ 2} + \phi_y^{\ 2} + \alpha\phi\ddot{\phi})dA - \int_s \phi\phi_n ds$$

Finite element approximation within an element is $\phi(x,y,t) = \sum_{i-1}^{n} N_i(x,y)\phi_i(t)$ where nodal variable ϕ are function of time. Stationary of above functional at any time 't' will result in the following final discretized form of the equation:

$$[H]\{\phi\} + [C]\{\ddot{\phi}\} + \{F\} = \{0\}$$

in which $[H]=\sum[H^e]$, $[C]=\sum[C^e]$ and $[F]=\sum[F^e]$.

- **a)** What would be the continuity requirements for finite element approximation?
- **b)** Derive matrics $[H^e]$, $[C^e]$ and $[F^e]$.
- **c)** Using **one** of the several methods of direct integration, other than those described in the class and lecture note, describe step-by-step procedure to solve the above equation. Also note on stability of the method.

2) You are asked to analyze the problem of seepage under the structure shown in the figure.

- **a)** A review of the seepage theory is needed to be provided in less than two pages. What is the variational form of the problem? Obtain the field equation and the associated boundary conditions.
- **b)** Choose an element of your choice and formulate the finite element problem at hand in detail.
- **c)** Describe the solution procedure for the problem at hand to obtain the field variable.
- **d)** How would you determine the pressure distribution along the bottom of the structure and hence calculate the seepage force causing uplift on the structure (Bernoulli's Theoreme).