Numerical Evaluation of Dynamic Response – Nonlinear SDF Systems

Advanced Structural Dynamics

M Ahmadizadeh, PhD, PE

Sharif University of Technology
Department of Civil Engineering

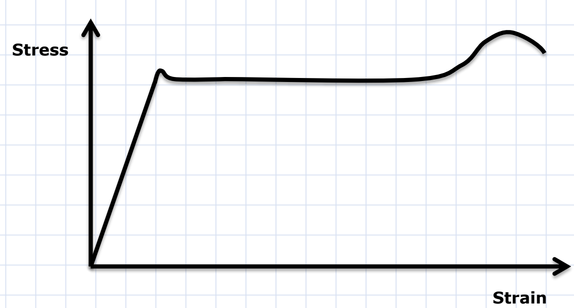
Contents

Scope and Goals

- Single-Degree-of-Freedom Systems
- Essentially-Strain Dependent Nonlinearity – Mostly Ductile
- Getting Familiar with the Essentials of Nonlinear Analysis
- Getting Familiar with Parameters that Govern the Nonlinear Behavior
- Understanding the Dynamic Response of Nonlinear Systems and the Effects of Dynamic Properties on the Response

Modeling of Nonlinear Behavior

- Elastic-Perfectly Plastic Behavior
  - The first nonlinear behavior to learn in Structural Engineering
  - Idealized based on the behavior of a ductile material (e.g. steel) before reaching strain hardening



Modeling of Nonlinear Behavior

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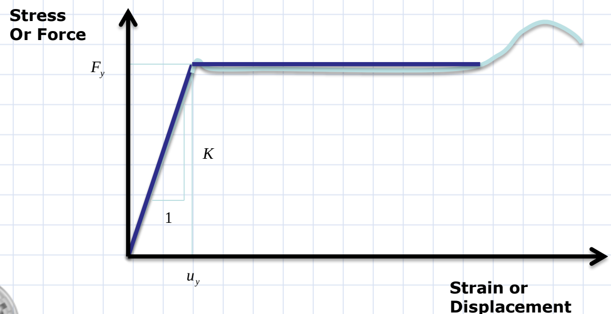
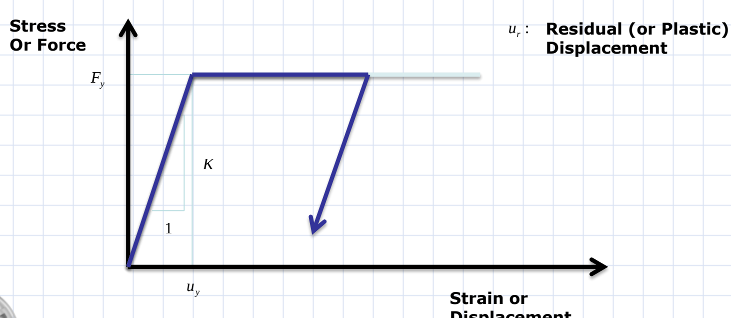


Modeling of Nonlinear Behavior

- Elastic-Perfectly Plastic Behavior
  - Easy to formulate and analyze

$$f_s(u)=r(u)=\begin{cases}K(u-u_r) & K(u-u_r)<F_y\\ F_y & \text{otherwise}\end{cases}$$

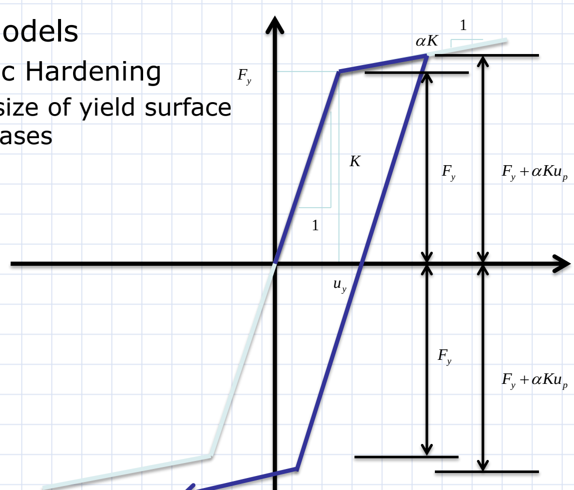


Modeling of Nonlinear Behavior

- Bilinear Models
  - Based on the more realistic assumption that not all section fibers yield simultaneously, or the material shows some post-yield stiffness
  - More stable analysis, since the stiffness is not suddenly lost
  - In addition to yield conditions (more formally, the yield surface, when multi-axial stress conditions exist), we need to decide how the behavior evolves after yielding
    - Isotropic Hardening
    - Kinematic Hardening

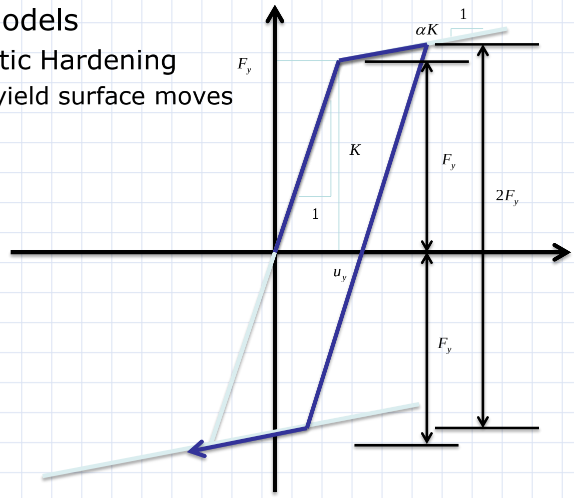
Modeling of Nonlinear Behavior

- Bilinear Models
  - Isotropic Hardening
    - The size of yield surface increases



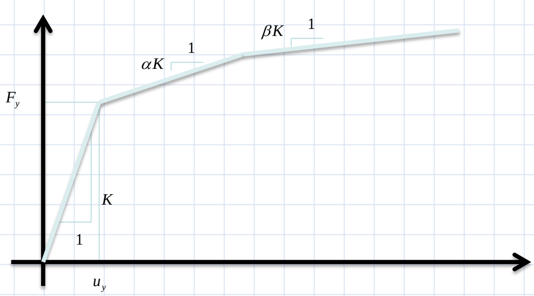
Modeling of Nonlinear Behavior

- Bilinear Models
  - Kinematic Hardening
    - The yield surface moves



Modeling of Nonlinear Behavior

- Tri-linear and Multi-linear Models
  - Multiple yield surfaces



  - Example: plastic hinge moment/rotation diagram

Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - An all-in-one model to represent a variety of nonlinear behaviors

$$r(t)=\alpha ku(t)+(1-\alpha)ku_y z(t)$$

$$\dot{z}(t)=\frac{1}{u_y}\left[\dot{u}(t)-\gamma\left|\dot{u}(t)\right|z(t)\left|z(t)\right|^{n-1}-\beta\dot{u}(t)\left|z(t)\right|^{n}\right]$$

  - Requires to keep track of an additional parameter
  - Can be solved using the same approach used for the overall structural analysis

Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - Example:

$\alpha=0$

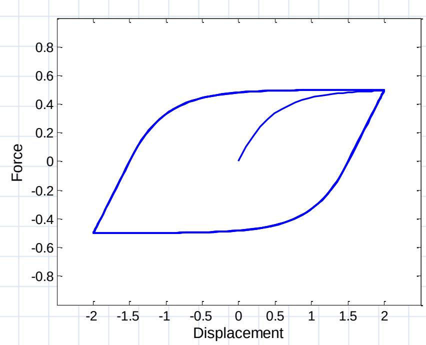
$\beta=1.0$

$\gamma=1.0$

$k=1.0$

$u_y=1.0$

$n=1.0$



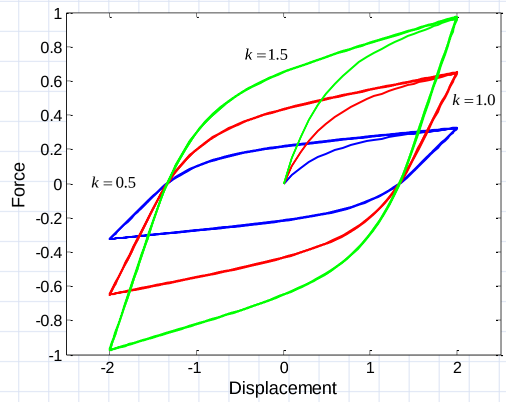
  - Note that this model is not rate-dependent
  - Figure shown using a static sinusoidal displacement

Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - Effect of Bouc-Wen Model Parameters: k

$\alpha = 0.1$
$\beta = 1.0$
$\gamma = 1.0$

$u_y = 1.0$
$n = 1.0$

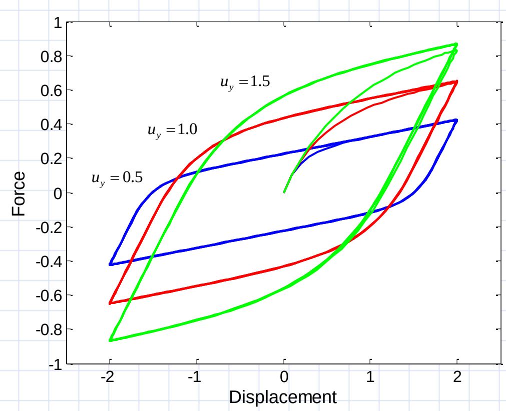


Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - Effect of Bouc-Wen Model Parameters: u_y

$\alpha = 0.1$
$\beta = 1.0$
$\gamma = 1.0$
$k = 1.0$

$n = 1.0$



Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - Effect of Bouc-Wen Model Parameters: α

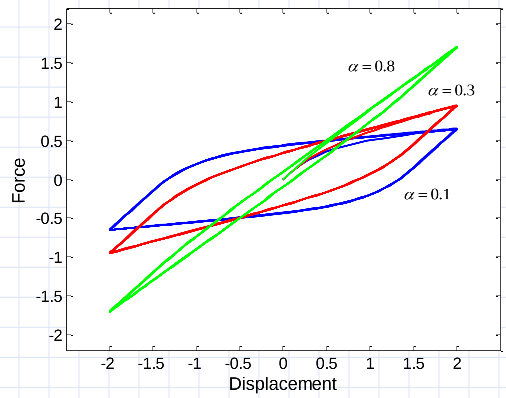
$\beta = 1.0$

$\gamma = 1.0$

$k = 1.0$

$u_y = 1.0$

$n = 1.0$



Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - Effect of Bouc-Wen Model Parameters: β

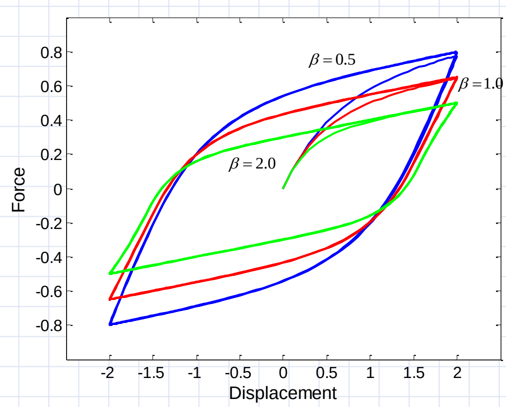
$\alpha = 0.1$

$\gamma = 1.0$

$k = 1.0$

$u_y = 1.0$

$n = 1.0$



Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - Effect of Bouc-Wen Model Parameters: γ

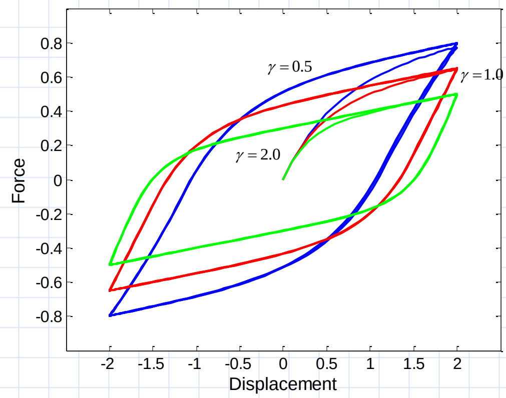
$\alpha = 0.1$

$\beta = 1.0$

$k = 1.0$

$u_y = 1.0$

$n = 1.0$



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Modeling of Nonlinear Behavior

- Bouc-Wen Model
  - Effect of Bouc-Wen Model Parameters: n

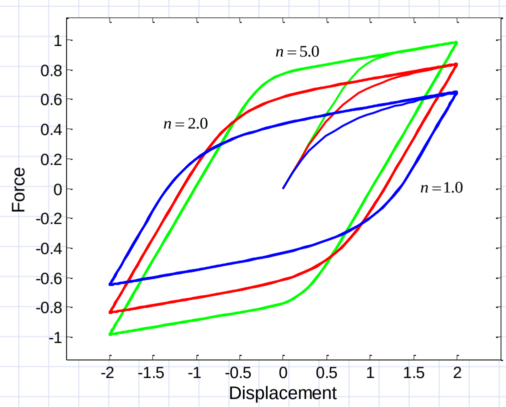
$\alpha = 0.1$

$\beta = 1.0$

$\gamma = 1.0$

$k = 1.0$

$u_y = 1.0$



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Modeling of Nonlinear Behavior

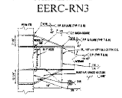
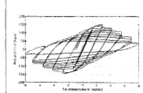
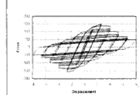
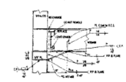
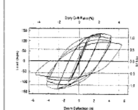
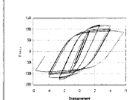
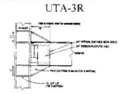
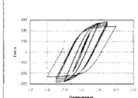
- Bouc-Wen Model
  - Selection of suitable parameters is carried out by directly comparing the model outcome to the desired force-displacement relationship (e.g. an experimentally obtained relationship)
  - Bouc-Wen Model Assumes Kinematic Hardening

Modeling of Nonlinear Behavior

- Other Models
  - More advanced models exist that include the nonlinearities resulting from gaps and degradation. For example, refer to:
    - Sivaselvan and Reinhorn, Hysteretic Models for Deteriorating Inelastic Structures, ASCE JSE 126(6), 633-640.

Modeling of Nonlinear Behavior

- Other Models
  - Sivaselvan and Reinhorn

SPECIMEN	EXPERIMENTAL RESPONSE	ANALYTICAL RESPONSE	PARAMETERS	
EERC-RN3  Top and bottom triangular haunch, Beam flange not welded. Flange local buckling, Web Distortion			α	10
			β_1	0.60
			β_2	0.30
			μ_{ult}	8.0
UCSD-1R  Bottom Haunch, Unconnected beam bottom flange, Local and lateral torsional buckling, Haunch yielding			α	5
			β_1	0.40
			β_2	0.20
			μ_{ult}	7.1
UTA-3R  Top and bottom triangular haunch, Web doubler plate, Flange and web buckling, Specimen twisting			α	4
			β_1	0.20
			β_2	0.30
			μ_{ult}	5.6

Dynamic Behavior of SDF Systems

- Parameters Affecting the Nonlinear Response of SDF Systems

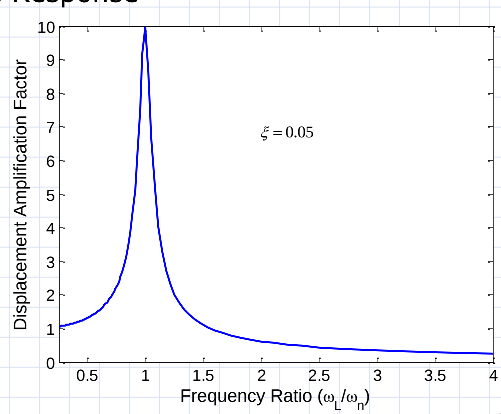
- Initial Stiffness
- Mass
- Damping
- Loading Characteristics
- Yield Displacement
- Ductile/Brittle Behavior
- Post-Yield Stiffness
- Ductility Demand (Share of Post-Yield Behavior in Response)

Dynamic Behavior of SDF Systems

- Parameters Affecting the Nonlinear Response of SDF Systems
- Natural Frequency (Initial)
- Damping Ratio
- Loading Characteristics
- Yield Displacement
- Ductile/Brittle Behavior
- Post-Yield Stiffness
- Ductility Demand (Share of Post-Yield Behavior in Response)

Dynamic Behavior of SDF Systems

- Dynamic Amplification Factors – Linear Response
  - Harmonic Loading
  - Steady-State Response



Dynamic Behavior of SDF Systems

- Dynamic Amplification Factors – Nonlinear Response
  - Need to include additional parameters (yield displacement, post-yield stiffness, hysteretic behavior, etc) in analysis
  - Results usually obtained for elastic-perfectly plastic systems to reduce the involved parameters and analysis costs
  - Here, we use a simple Bouc-Wen model to study the response characteristics of nonlinear SDF systems

Dynamic Behavior of SDF Systems

- Dynamic Amplification Factors – Nonlinear Response
  - Model Properties:
    - Common Parameters

$m=1 \qquad \xi=0.05$

$\beta=1 \qquad \gamma=1 \qquad n=1$

    - Variable Parameters

$k=10$

$u_y=1$

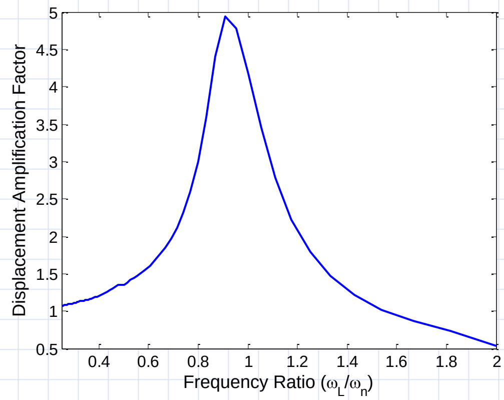
$\alpha=0$

Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 1.00$

$\alpha = 0$



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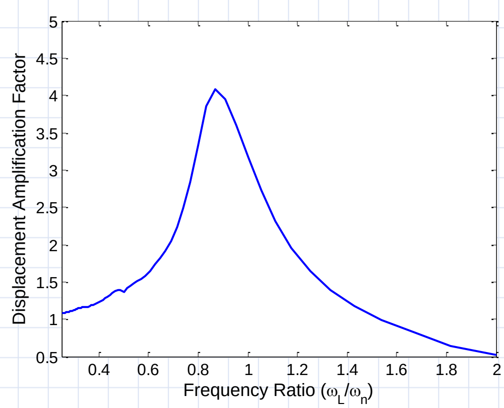
Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.50$

$\alpha = 0$



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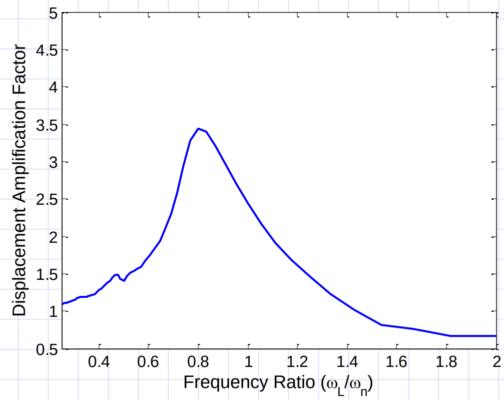
Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.25$

$\alpha = 0$

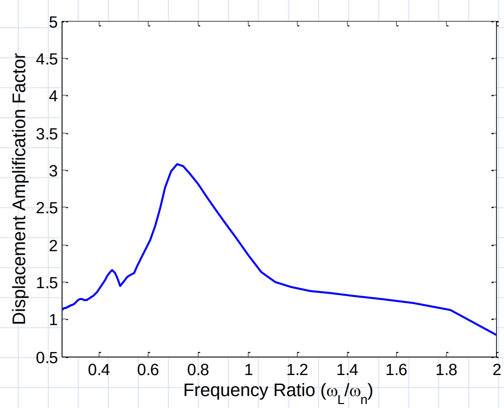


Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.12$

$\alpha = 0$

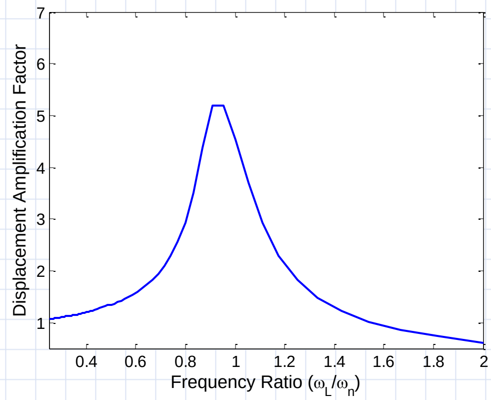


Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 1.00$

$\alpha = 0.20$

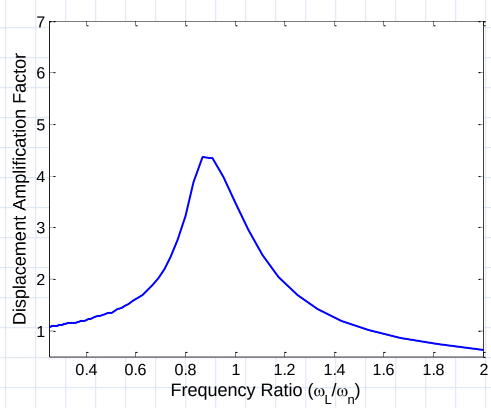


Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.50$

$\alpha = 0.20$

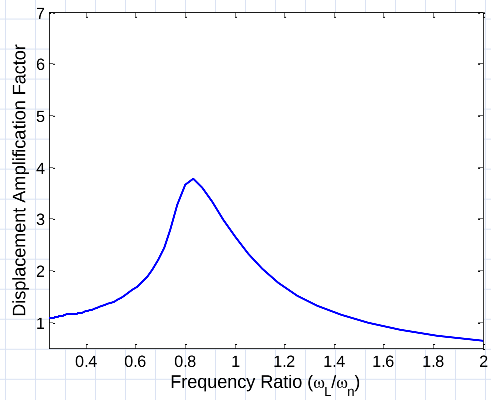


Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.25$

$\alpha = 0.20$

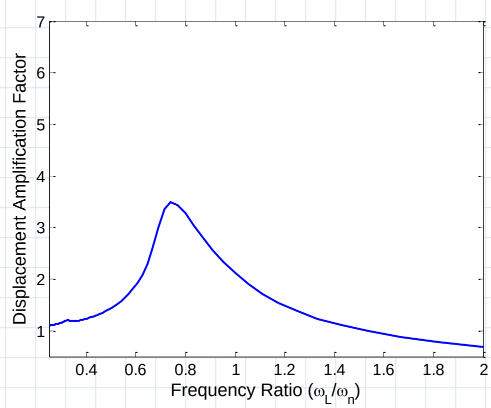


Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.12$

$\alpha = 0.20$

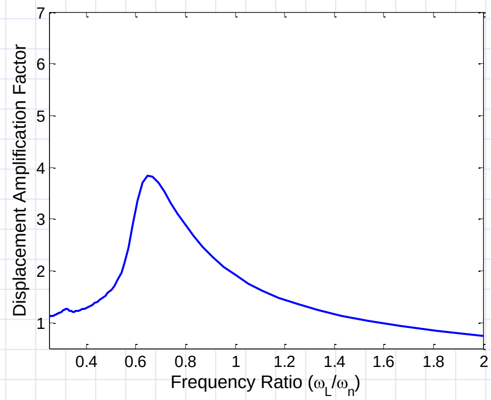


Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.06$

$\alpha = 0.20$

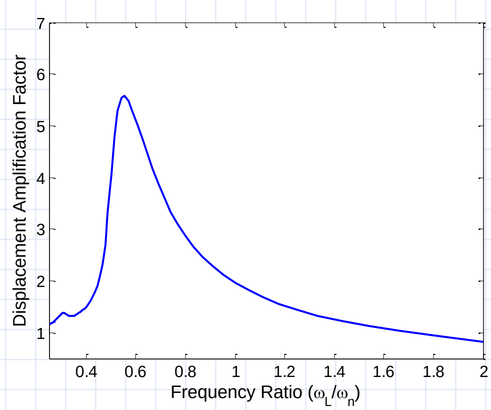


Dynamic Behavior of SDF Systems

- Effect of Yield Displacement

$u_y = 0.03$

$\alpha = 0.20$

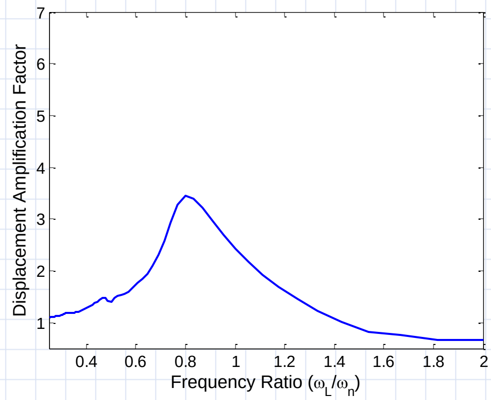


Dynamic Behavior of SDF Systems

- Effect of Post-Yield Stiffness

$u_y = 0.25$

$\alpha = 0.00$

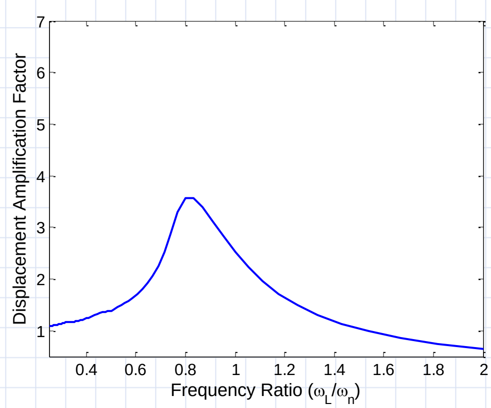


Dynamic Behavior of SDF Systems

- Effect of Post-Yield Stiffness

$u_y = 0.25$

$\alpha = 0.10$

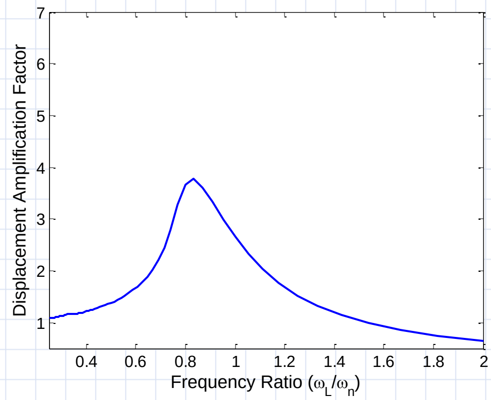


Dynamic Behavior of SDF Systems

- Effect of Post-Yield Stiffness

$u_y = 0.25$

$\alpha = 0.20$

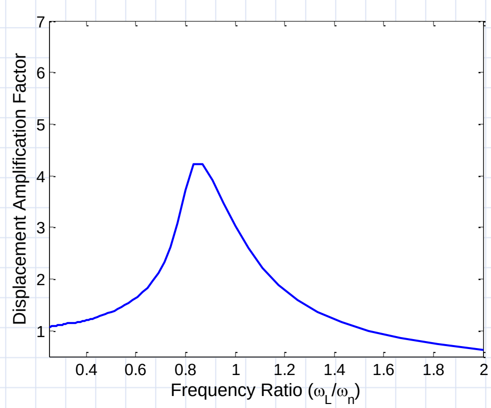


Dynamic Behavior of SDF Systems

- Effect of Post-Yield Stiffness

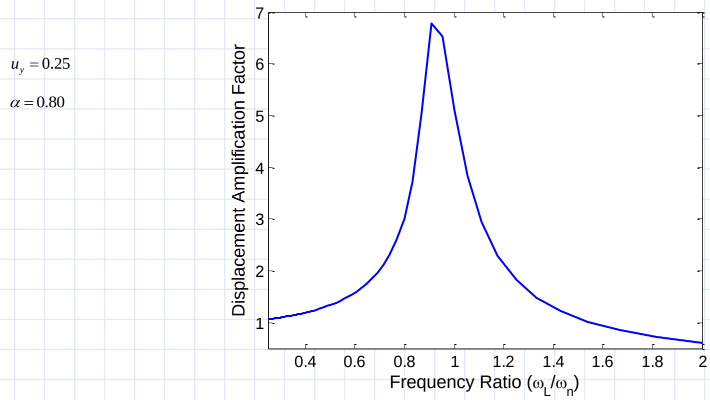
$u_y = 0.25$

$\alpha = 0.40$



Dynamic Behavior of SDF Systems

- Effect of Post-Yield Stiffness



Dynamic Behavior of SDF Systems

- Response Spectra
  - Parameters in linear spectra:
    - Natural Frequency
    - Damping Ratio
  - Parameters in nonlinear spectra:
    - Natural Frequency (initial)
    - Damping Ratio
    - Ductility
      - » Results in more detailed analysis needs, since ductility is not known before the analysis
    - Note that the vertical axis in a nonlinear spectrum is used to determine the yield acceleration and yield strength; i.e. the yield strength to achieve the given ductility

Dynamic Behavior of SDF Systems

- Developing Nonlinear Response Spectra
  - For a Given Ductility Ratio
    - For Each Natural Frequency and Damping Ratio

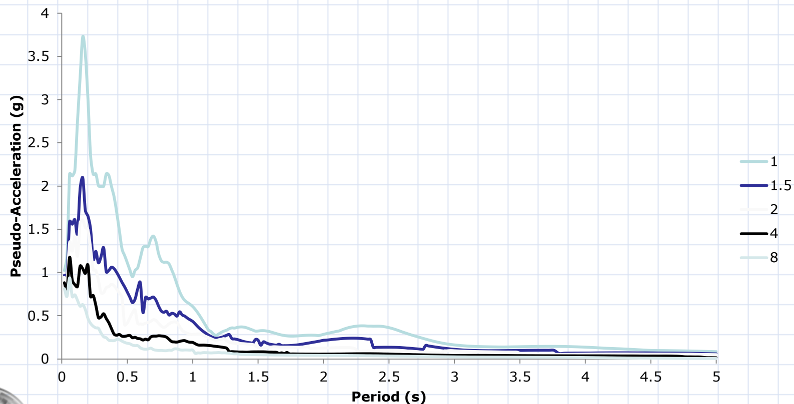
      1. First assume a yield displacement
      2. Obtain the nonlinear response
      3. Calculate ductility
      4. Compare obtained ductility with the desired value
         - » If the ductility is close enough, record the maximum response as the point of the spectrum corresponding to the required frequency, damping ratio and ductility
         - » Otherwise, adjust the yield displacement and repeat from Step 2

Dynamic Behavior of SDF Systems

- Developing Nonlinear Response Spectra
  - This usually requires many analyses with small time steps. This is particularly problematic in stiff systems (systems with large natural frequency), since in these systems, the post-yield displacements are usually very large. Obtaining the desired ductility ratio may require very high precision in the analysis.
  - Usually, the precision is not kept constant for the entire range of frequencies

Dynamic Behavior of SDF Systems

- Nonlinear Response Spectra – Example: Abbar Longitudinal Record



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