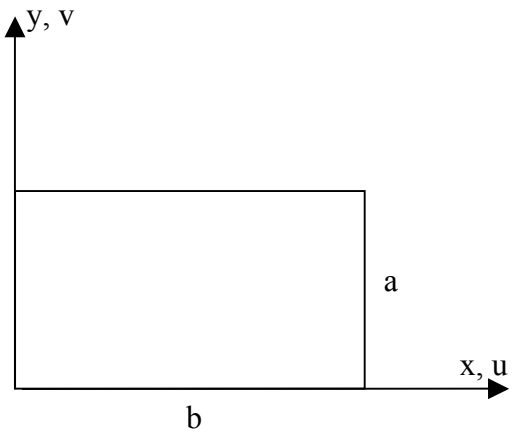
Assignment Number 3

1 Changes in temperature variation within a plane rectangular element is assumed to be linear (i.e. nodal temperature changes T_1^0, T_2^0 , T_3^0 and T_4^0 are known). How would you incorporate this changes in a thermal elastic analysis? Derive the load vector required to incorporate a temperature change. How would you calculate the element stresses?

2. Compute the plane strain stiffness matrix in terms of the ration r = a/b for the rectangular element shown. The element has unit thickness. Use the following displacement model. Specialize the matrix for ν = 0.2, r=1

$$u(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy$$
$$v(x,y) = \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy$$



$$[k] = \begin{bmatrix} 22 & -13 & 2 & -11 & 7.5 & -1.5 & 1.5 & -7.5 \\ & 22 & -11 & 2 & 1.5 & -7.5 & 7.5 & -1.5 \\ & & 22 & -13 & -1.5 & 7.5 & -7.5 & 1.5 \\ & & & 22 & -7.5 & 1.5 & -7.5 & 1.5 \\ & & & & 22 & 2 & -13 & -11 \\ & & \text{Symmetric} & & & 22 & -11 & -13 \\ & & & & & & 22 & 2 \\ & & & & & & & 22 \end{bmatrix} \quad \nu=0 \text{ and } r=1$$

3. Introduce four secondary external nodes (one per side) to the rectangular element in problem no.2 above and obtain its stiffness equations. Adopt the following quadratic displacement model:

$$u(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^2 y + \alpha_8 xy^2$$
$$v(x,y) = \alpha \ \ + \alpha_{10} x + \alpha_{11} y + \alpha_{12} x^2 + \alpha_{13} xy + \alpha_{14} y^2 + \alpha_{15} x^2 y + \alpha_{16} xy^2$$