Optimal sensor placement for source separation with noisy measurements

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EUSIPCO2023
31ST EUROPEAN SIGNAL PROCESSING CONFERENCE
SEPTEMBER 4–8, 2023 • HELSINKI, FINLAND

Introduction

Objective: Identify optimal sensor positions for efficient source separation from noisy mixed signals.

- Optimal sensor placement is important in areas like structural health monitoring, water networks, and wireless sensor networks.
- The presence of noise magnifies the influence of sensor positions on linear source separation quality.
- The proposed placement criterion, based on the SINR of the separated signals, enhances separation quality and outperforms previous criteria in single source extraction.
- Spatial gains (signal attenuation) are modeled statistically, and a method for estimating them using BSS techniques is discussed.

Problem statement

Linear mixing model

$$\mathbf{y}(\mathbf{X}_M, t) = \sum_{p=1}^{P} \mathbf{a}_p(\mathbf{X}_M) s_p(t) + \mathbf{n}(\mathbf{X}_M, t)$$

$\mathbf{y}(\mathbf{X}_M, t) \in \mathbb{R}^M$: Vector of the measurements for M sensors located at $\mathbf{X}_M = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_M\}$.
$s_p(t)$: The p-th source signal.
$\mathbf{a}_p(\mathbf{X}_M)$: Vector of spatial gains (attenuation coefficients) at the sensor positions.
$\mathbf{n}(\mathbf{X}_M, t)$: Additive noise at the sensor positions with the known covariance matrix $\mathbf{C}^n_{MM}$.

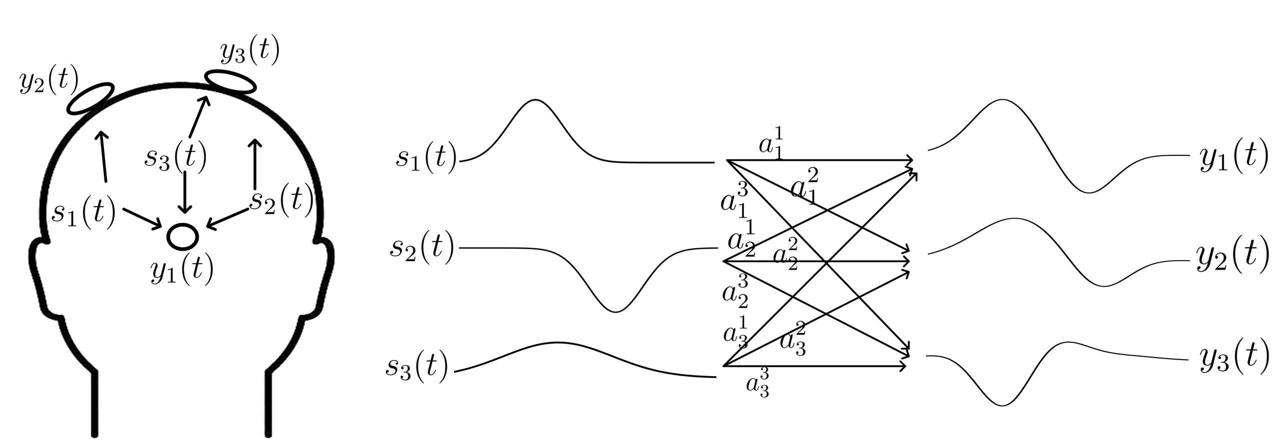


Fig. 1: Example of linear mixing model: EEG recordings.

Consider the vector $\mathbf{f}_l \in \mathbb{R}^M$ used to linearly estimate the l-th source : $\hat{s}_l(t) = \mathbf{f}_l^T \mathbf{y}(\mathbf{X}_M, t)$. The optimum vector $\mathbf{f}_l^*$ that maximizes the SINR of the estimated source is given by

$$\mathbf{f}_l^* = \left(\sum_{p=1, p\neq l}^{P} \mathbf{a}_p(\mathbf{X}_M)\mathbf{a}_p(\mathbf{X}_M)^T + \mathbf{C}^n_{MM} \right)^{-1} \mathbf{a}_l(\mathbf{X}_M),$$

and the maximum achievable SINR at the sensor positions is obtained as

$$\mathrm{SINR}_l(\mathbf{f}_l^*, \mathbf{X}_M) = \mathbf{a}_l(\mathbf{X}_M)^T \left(\sum_{p=1, p\neq l}^{P} \mathbf{a}_p(\mathbf{X}_M)\mathbf{a}_p(\mathbf{X}_M)^T + \mathbf{C}^n_{MM} \right)^{-1} \mathbf{a}_l(\mathbf{X}_M).$$

Proposed sensor placement criterion: sum of the optimum SINRs of all sources.

$$J(\mathbf{X}_M) = \sum_{l=1}^{P} \mathbb{E}\left[\widehat{\mathrm{SINR}}_l(\mathbf{f}_l^*, \mathbf{X}_M) \right]$$

Statistical modelling of the spatial gains

- Spatial gains in practical use not fully known per space.
- Uncertainty due to limited measured points and errors.
- Gaussian process used to model spatial gains considering prior info and uncertainty.

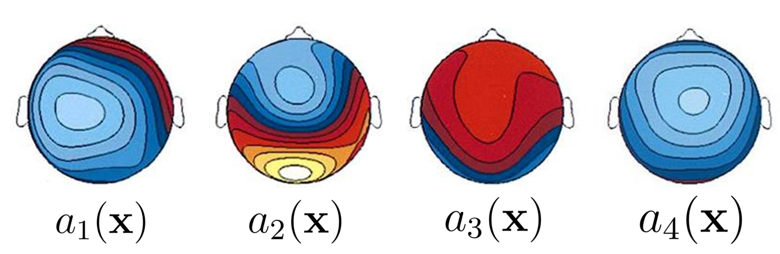


Fig. 2: Distribution of the spatial gains of 4 source components in an EEG experiment [1]

$$\hat{a}_p(\mathbf{x}) \sim \mathcal{GP}\left(m^{a_p}(\mathbf{x}), C^{a_p}(\mathbf{x}, \mathbf{x}')\right)$$

$m^{a_p}(\mathbf{x})$: Mean of the stochastic process, containing the prior knowledge.
$C^{a_p}(\mathbf{x}, \mathbf{x}')$: Kernel function specifying the covariance between the points in space.

Optimization Problem

How to choose optimum placement positions?

- $\mathbf{X}_T$: the set of T candidate points on a grid, M points should be chosen.
- Combinatorial search over $\frac{T!}{M!(T-M)!}$ possibilities required : impractical due to computation time.

Greedy Method

Step-by-step sensor placement.
- Optimize the position of one sensor at each step.
- Previously placed sensors remain fixed.
$\Rightarrow$ Provides near-optimal solutions with computational efficiency.

Sequential Approach

In the step by step placement, use measurements of previously placed sensors to estimate spatial gains and update the stochastic model.

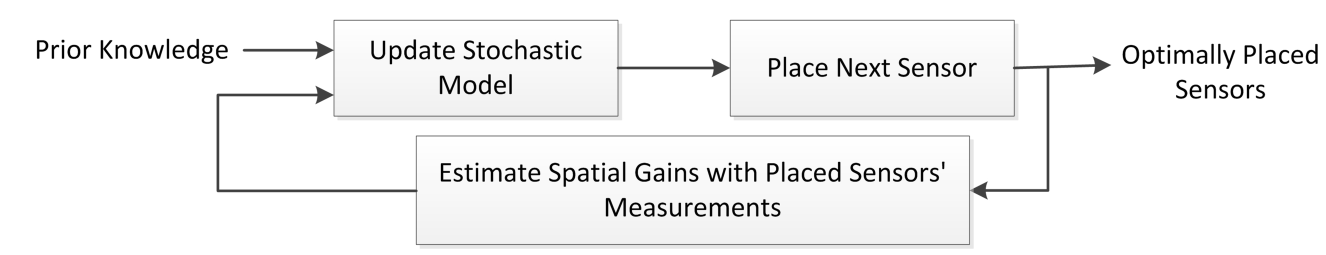


Fig. 3: Sequential Approach

BSS estimation technique for spatial gains:

- Already placed sensors are used to measure mixed signals.
- PCA is used for dimension reduction along with noise cancellation.
- FastICA is applied to separate sources from noisy measurements.
- Estimation of spatial gains is performed at sensor positions.

Numerical Results

Comparison with previous expected SNR criterion [2]. New criterion outperforms in numerical results.

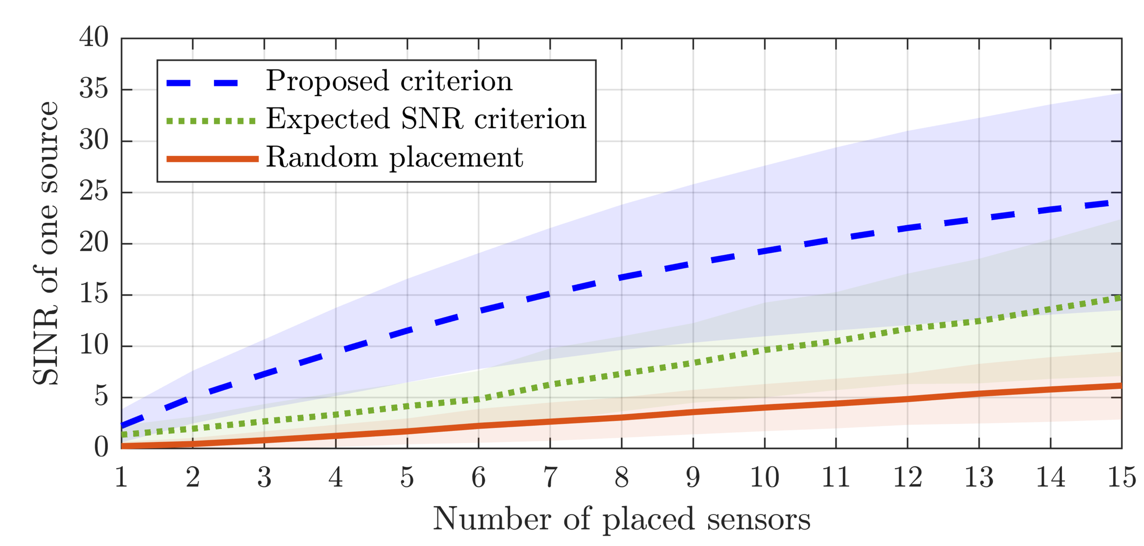
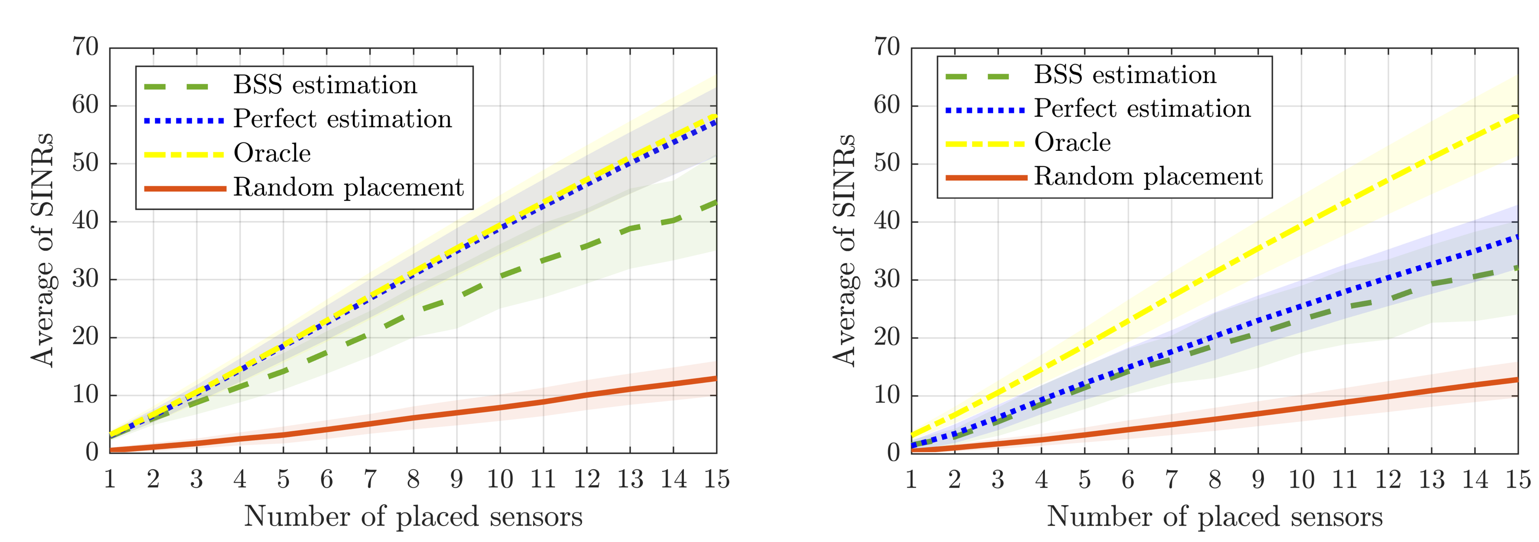


Fig. 4: Single source extraction: comparison of proposed and expected SNR criteria

In source separation, several scenarios were compared: the sequential approach with perfect and BSS estimation of the spatial gains, random placement, and the oracle (perfect prior knowledge).



(a) Low level of uncertainty (b) High level of uncertainty

Fig. 5: Performance of sensor placement method in source separation

Conclusion

- Proposed SINR-based criterion outperforms prior criterion for source extraction.
- Enhancement in source separation quality via proposed placement method depends on the available knowledge about the spatial gain.
- Effective utilization of BSS spatial gain estimation to update stochastic model.

[1] M. Scott et al., "Blind separation of auditory event-related brain responses into independent components," *Proc. Natl. Acad. Sci. U.S.A.*, vol. 94, n. 20, pp. 10974–10984, 1997.
[2] F. Ghayem, B. Rivet, C. Jutten, and R. C. Farias, "Optimal sensor placement for signal extraction," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.* IEEE, 2019, pp. 4978–4982.