A Novel Impulsive Noise Cancellation Based on Successive Approximations

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Abstract

In this paper we will propose a new method to recover lowpass signals corrupted by impulsive noise. The new method uses adaptive thresholds in conjunction with soft-decision and successive approximations to find the position and values of all the impulses that the redundancy of the signal allows. Computer simulations confirm the robustness of the proposed algorithm when corrupted samples exceed reconstruction capacity.

1. Introduction

Impulsive noise is a common phenomenon occurring in channels that suffer from switching, manual interruptions and lightning. In such channels several samples of the signal (sparse or burst) are lost. To recover the original signal, redundancy is introduced at the transmitter, for example by inserting zeros in the DFT domain (over sampling). Many practical signals such as speech and image are already band-limited; i.e., the high frequency DFT (or DCT) coefficients are almost zero. We can take advantage of this redundancy for error concealment and impulsive noise removal.

By definition, in an erasure channel, the locations of losses are known. Error recovery in an erasure channel have been studied extensively by [1, 2]. The problem of impulsive noise, where locations of errors are not known, have been studied in several papers using DFT codes [3, 4, 5]. In [4], a decoding technique for DFT-based error control codes, based on error locator polynomial is devised. DFT codes are essentially equivalent to Reed-Solomon(RS) codes in real/complex fields. Thus it has all the properties of RS codes such as Maximum Distance Separable (MDS). The method proposed in this paper is robust to overloaded. By overloaded impulsive noise we mean a situation which the number of corrupted samples exceeds the reconstruction capacity of the DFT code.

In this paper a method similar to the radar's Constant False Alarm Rate (CFAR) [6] is used to generate adaptive thresholds which are used in to detect impulsive noise. Instead of using hard-decision to detect the locations, a soft-decision method is used. After detecting the locations of the impulses, the amplitude of the corrupted samples are estimated. This estimate is used successively in a loop to improve the quality of signal reconstruction. This method is called Iterative Detection Estimation (IDE) [7] and is used in Sparse Component Analysis (SCA) applications.

The input of the detection block is an estimate of impulsive noise calculated by subtracting estimated signal from the received noisy signal. The output of the detection block (which is called *mask*) contains information about the locations of the impulses. This output is used in the estimation block to estimate the signal. In the estimation block we use an iterative method used in missing sample problems, because by detecting the position of impulses, we have an erasure problem that can be solved by the iterative method introduced in [8]. As mentioned earlier the estimated signal is used in a loop to improve the noise estimate. In this paper we call each of these detection and estimation steps an IDE step. As the number of IDE steps increase, a better estimate of signal is calculated. At the same time, when better estimates of the signal is devised, the soft-decision method gradually is pushed towards hard-decision (by using a parameter called α described in later sections) in order to make more accurate decisions about whether a sample is noisy or not.

The proposed method efficiently uses nearly all the redundancy introduced in the signal, thus its reconstruction capacity (the number of corrupted samples that are recovered) tends to the theoretical limit (which is half of the inserted zeros as described later). The successive use of noise detection and the amplitude estimation are portrayed in Fig. 1.

2. Detection

The received signal contains impulsive noise added to the original lowpass signal. To be specific, we use the term impulsive noise, for a discrete signal with N samples which only M of them are random variables with Gaussian PDF and other samples are zero. The locations of these M nonzero samples are taken to be uniformly distributed between the N samples. Detection is performed according to the amplitude of the estimated noise. The noise estimate is calculated by subtracting estimated signal (in the previous IDE step) from the received

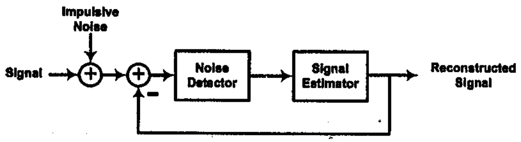

Figure 1: Iterative use of noise detector and signal estimator (IDE).

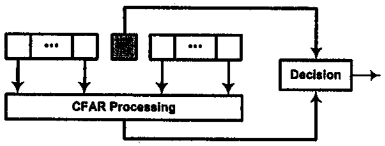

Figure 2: CFAR thresholding and decisioning blocks.

signal. Depending on the relative amplitudes of signal and noise, the detector can make two kinds of mistakes. A *missed detection* occurs if a corrupted sample is not detected, while a *false alarm* occurs if a legitimate sample is detected as noise. In the proposed detection block a combination of CFAR thresholding and soft-decision is used to decrease these errors. As mentioned earlier we use the term mask for the output of the detection block. This signal is used in the estimation block to attenuate impulses while have little effect on legitimate samples. In the estimator each sample of the signal is multiplied by the corresponding sample of the mask to eliminate the impulses. Thus if the i^{th} sample of an ideal mask, m_i is defined as

$$m_i = \begin{cases} 0 & r[i] \text{ is noisy} \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

where $r[i]$ is the i^{th} sample of the received signal.

2.1 Adaptive Detection Vs. Non-adaptive Detection

Detection can be either adaptive or non-adaptive. In the non-adaptive method, a fixed threshold is used to compare the amplitude of the sample under test to determine whether the signal at that instant is contaminated with impulsive noise or not. In the adaptive detection, the sample under test is compared with an adaptive threshold which is determined according to the amplitude of the adjacent samples. When the statistical parameters of the noise is not known, adaptive thresholding techniques based on Neyman-Pearson criterion are used in radar detection to maintain a Constant False Alarm Rate(CFAR). Figure 2 depicts a simple CFAR detector.

The CFAR processing is combination of adjacent samples (reference cells). In this paper we used a Censored Mean Level (CML) CFAR. In an k^{th} order CML-CFAR of length n, k of the smallest reference cell amplitudes are averaged and the other $n-k$ samples are ignored. The CML-CFAR is used to discard impulsive noises present in the reference cells, which otherwise would increase the adaptive threshold unnecessarily.

2.2 Hard-decision Vs. Soft-decision

The detection process is not an error-free process in the early stages, specially when the noise level is near the signal level. Thus if the estimator is flexible, instead of hard-decision, a soft-decision method can be used. The soft-decision can tend to hard-decision when better estimates of the original signal and impulsive noise is produced. Using hard-decision within the detection block a two-state mask is generated; i.e, zero if it detects an impulsive noise and one otherwise. The soft-decision block generates a real number between zero and one depending on the certainty of the detector. Simulation results suggest that a function of the form

$$\phi(x) = e^{-\alpha|x|} \tag{2}$$

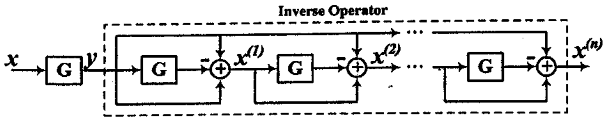

Figure 3: Block diagram of the iterative method with $\lambda = 1$

performs well if it is used to generate mask. In (2), x is the difference between the sample under scrutiny and the threshold generated by CFAR. $\phi(x)$ tends to one if the amplitude of x tends to zero (there is a little difference between a sample and the neighbouring samples) and it tends to one if x is made large. Parameter α determines the slop of ϕ at a given x. If α is made larger, even when x is small, the mask tends to zero, thus the soft-decision tends to hard-decision.

3. Estimation Using an Iterative Method

Estimating corrupted samples is possible because of the redundancy present in the original signal or introduced in the signal (in the transmitter). In this paper, oversampling is used to introduce redundancy in the signal. This is equivalent to padding zeros in the DFT domain. If N_z zeros are padded in the DFT domain, the receiver should be able to reconstruct N_z lost samples(where the locations of losses is known). In the denoising problem, locations of corrupted samples, i.e., impulsive noise is not known to the receiver, hence the locations of corrupted samples need to be determined before estimating the signal. This doubles the number of unknown variables for the receiver. Thus for the denoising problem, ideally the receiver should be able to detect and reconstruct $N_z/2$ corrupted samples.

3.1 An Iterative Method

The iterative method introduced in [2, 9] is general approach to approximate the inverse of a system. The system can be non-linear and/or time varying. If G is a distortion operator representing the system and $y = Gx$ is known, then the objective is to reconstruct x. Thus, synthesizing G^{-1} is aimed. Symbolically, G^{-1} is $\lambda \sum_{i=0}^{\infty}(I - \lambda G)^i$, thus the series

$$x_k = \lambda \sum_{i=0}^{k} (I - \lambda G)^i \cdot y \tag{3}$$

approaches x as k increases. A recursive relation between x_k and x_{k+1} (called the iterative method) is

$$x_{k+1} = x_k + \lambda G(x - x_k) \tag{4}$$

where λ is the relaxation parameter that determines the rate of convergence and k in the number of iterations. If λ is larger than a threshold, the iterative method diverges. It can be shown that this argument is true only if the norm of the operator $I - \lambda G$ is less than one; i.e., the energy of the signal is greater than the energy of the distortion error caused by the operator G. Figure 3 depicts the approximate inverse system, using the iterative method with λ set to one. The more the number of iterations used, the better the approximation of the inverse system is obtained [2]. It has been shown that for small number of iterations, the iterative method results in the pseudo-inverse of the distortion block which is a rough and stable approximate of the inverse system [10]. For larger number of iterations, the inverse system is obtained. If the system is not invertible or ill-conditioned, it is possible that the iterative method diverges or yields a wrong answer.

3.2 Estimation

In an erasure channel, the location of lost samples are known at the receiver, but in our problem the receiver does not know the locations of impulses. Thus a detection block is used to detect these locations. Then we can use the iterative method in the way it is used in an erasure channel (locations of missing samples are known). In the denoising problem, G is an distortion caused by mask and lowpass filtering as depicted in Fig. 4, thus we

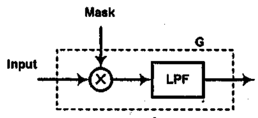

Figure 4: Distortion block (G)

refer to it as the distortion block. In estimator the mask generated by the detection stage is used to eliminate the corrupted samples by multiplying mask and noisy signal and the result is low-pass filtered. Then we use the iterative method depicted in Fig. 3. The result of the iterative method is an estimate of the signal. In the early IDE steps, by adjusting the number of iterations to a small number (about 20), a rough approximation of the signal (resulted from the pseudo inverse of distortion) is obtained. This approximation could be unstable if a large number of iterations is used. When better approximations of the signal is gained, then the number of iterations is set to a larger number, depending on the desired final SNR.

The iterative method is capable of using a mask that is generated with hard-decision or soft-decision detector. If hard-decision detector is used, the amplitude of each sample is compared with a threshold, and will result in zero if it is larger and one otherwise. Using soft-decision, the amplitude of estimated noise is subtracted from the threshold generated by CFAR and the result is applied to the exponential function mentioned in section 2.2.

Based on the characteristics of the channel, impulsive noise may distort consecutive samples. The iterative method may not be able to reconstruct the corrupted due to poor condition number of the matrix G. To handle this situation, instead of using DFT which uses $e^{2\pi/N}$ as its kernel, Sorted DFT (SDFT)[US patent no. 6 601 206] [11, 12] can be used which uses $e^{2\pi p/N}$ where p is a prime number. SDFT algorithm permutes the DFT coefficients. Thus SDFT acts as an interleaver and its inverse does the job of de-interleaving.

Based on the above discussions the proposed algorithm is as follows:

1. Calculate the noise estimate and generate thresholds using the CML-CFAR discussed in section 2.1
2. Generate a mask using soft-decision method introduced in section 2.2
3. Use the iterative method introduced in section 3 to estimate the signal.
4. Estimate the impulsive noise by subtracting the estimated signal from received signal.
5. Return to the first step. In the second step, increase α parameter of the exponential function to make soft-decision more alike to hard-decision. Increase the number of iterations used in the third step.

4. Simulation Results

To prove the effectiveness of the proposed method, different simulations were conducted. In all simulations the signal was a lowpass filtered white Gaussian pseudo-random signal. We also tested random signals with uniform distribution with very little difference.

In the first simulation, non-adaptive thresholding (using the same threshold for all of the samples of the signal) and hard-decision is used to detect the impulsive noise and generated the mask. The mask and noisy signal are multiplied and the result is filtered. This method does not use the redundancy present in the signal efficiently, but it can be viewed as a crude reconstruction algorithm. However, if this process is repeated as in Fig. 1, and we use more iterations in the estimation, we have improvement as portrayed in Fig. 5.

In the second simulation we use non-adaptive thresholding and soft-decisioning in the detection block to generate the mask. In the estimation block we used the iterative method (with 150 iterations) to estimate the signal. The number of corrupted samples in noisy signal is half of the number of inserted zeros in DFT domain (full capacity). In the third experiment CFAR thresholding is used instead of non-adaptive thresholding. The results are shown in Fig. 6. This figure suggests that by adding CFAR to the soft-decision detector, the recovery is enhanced.

If the channel impulsive noise exceeds the maximum capacity of the reconstruction algorithm, the SNR of the reconstructed signal is degraded gracefully. Fig. 7 depicts the SNR of the reconstructed signal when the errors

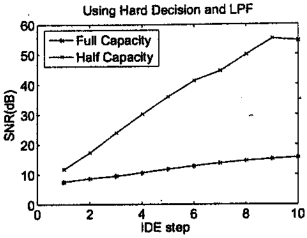

Figure 5: Multiple Detection-Estimation steps using hard-decision simple detector

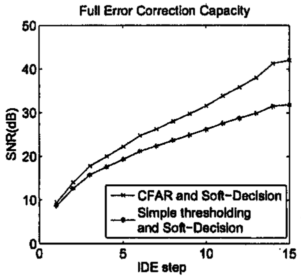

Figure 6: Comparison of CFAR and non-adaptive thresholding in conjunction with soft-decision

introduced by the impulsive channel is 15% more than the theoretical reconstruction capacity. It can be seen that the reconstruction SNR degrades compared to that of Fig. 6 but the algorithm does not diverge.

5. Conclusions

In this paper a new method for impulsive noise cancellation is proposed. The method could reach the theoretical upper bound of reconstruction capacity (MDS codes for real/complex Galois fields). We used a method call Iterative Detection Estimation (IDE) to use the estimated signal recursively in order to improve the reconstruction. In the detection block, CFAR thresholding and soft-decisioning are used to generate a mask. This mask is used in conjunction with the iterative method in the estimation block to estimate the signal. The SNR of reconstructed signal can be improved by increasing the number of IDE steps and increasing the number of iterations in the estimation block. Simulation results show that the SNR of the reconstructed signal degrades gradually if the number of impulsive noise exceeds the theoretical reconstruction capacity.

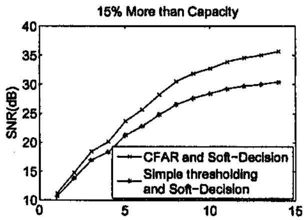

Figure 7: SNR of the denoised signal when the error rate exceeds the reconstruction capacity by about 15%

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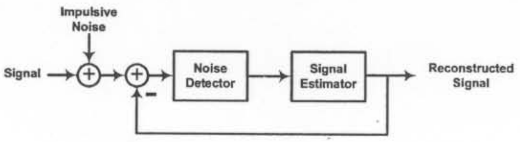

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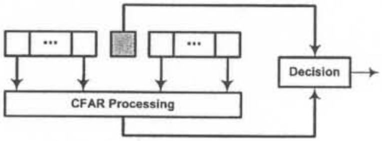

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