Introduction to Elementary Particle Physics

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Elementary Particle Physics

Lecture 12: Farvardin 17, 1398

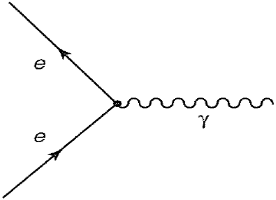
1397-98-II

Four forces:

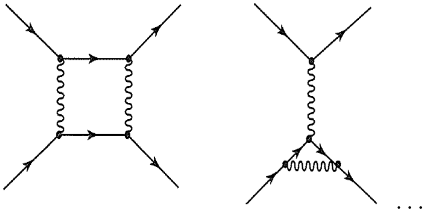
A. Quantum Electrodynamics (QED)

Feynman Rules of QED

$$\mathcal{L}_{QED}^{int} = -e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$



$$\mathcal{L}_{QED} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - e\bar{\psi}\gamma^{\mu}A_{\mu}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$



Feynman rules in QED

- The solid straight lines represent fermions (leptons and quarks in QED)
- The wavy lines represent bosons (photons in QED)
- An arrow indicating motion of a fermion forwards (backwards) in time is equivalent to a fermion (antifermion) moving forwards in time
- **External lines** represent real particles. They are **on mass-shell**
- **Internal lines** represent virtual particles. They are **off mass-shell**

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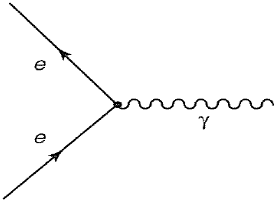
Feynman rules in QED

Primitive vertex of QED



- Fermions and boson lines meet at vertices
- At each vertex, the strength of the interaction is represented by a coupling constant *e*
- Each photon couples to fermions with a factor $\sqrt{\alpha} \sim e$
- At each vertex, charge and energy-momentum are conserved
- A diagram as a whole has also an energy-momentum conservation

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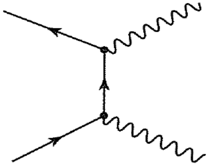
Remark 1:

- The primitive vertex of QED does not represent by itself a physical process, because of the following kinematic reasons

  a. $e^- \to e^- + \gamma$ would violate the conservation of energy. In the rest frame of electron, $E_e = m_e c^2$. It cannot decay into a photon and a recoiling electron (because the recoiling electron needs an energy greater than the rest energy of the electron $E_e = m_e c^2$).

  b. The reaction $e^- + e^+ \to \gamma$ is not kinematically possible, because in the center of mass frame the electron and the positron enter with equal and opposite velocities. Hence the total momentum at the vertex is zero. But the final momentum cannot be zero (photons always travel with the speed of light)

- The correct reaction should read: $e^- + e^+ \to 2\gamma$



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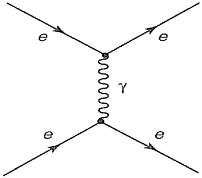
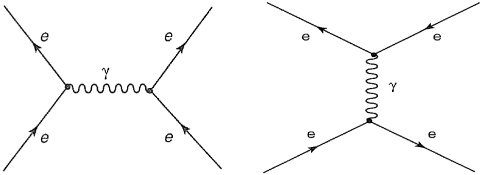
Important QED processes (Photon is the virtual mediator):

1. Møller Scattering: $e^- + e^- \to e^- + e^-$



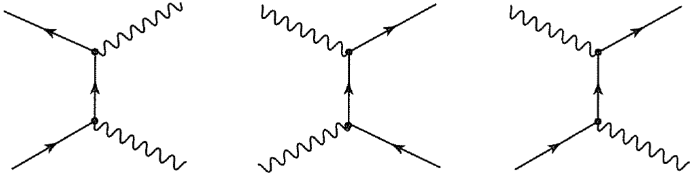
2. Bhabha Scattering: $e^- + e^+ \to e^- + e^+$



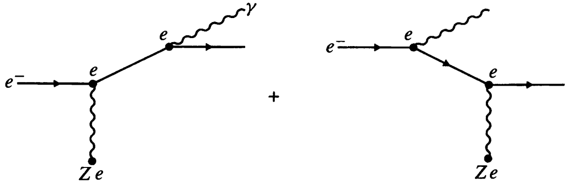
Here: Both diagrams are necessary to compute the **scattering amplitude** and eventually the **differential and total cross-sections** of the Bhabha scattering

Other important QED processes (Lepton is the virtual mediator):

3. Pair annihilation $e^- + e^+ \to 2\gamma$
4. Pair production $2\gamma \to e^- + e^+$
5. Compton scattering $e^- + \gamma \to e^- + \gamma$



6. : Bremsstrahlung process (counting the power of $\boldsymbol{\alpha}$)



- Emission of a real photon by an electron that has undergone acceleration in the electric field of a nucleus with charge *Ze*
- A virtual photon has to be exchanged with the nucleus in order to conserve momentum
- The amplitude is of order $\alpha^{3/2} \sim e^3$
- The cross-section$\sim$(amplitude)^2 is of order α^3
- An intermediate (virtual) electron is also involved (kinematic)

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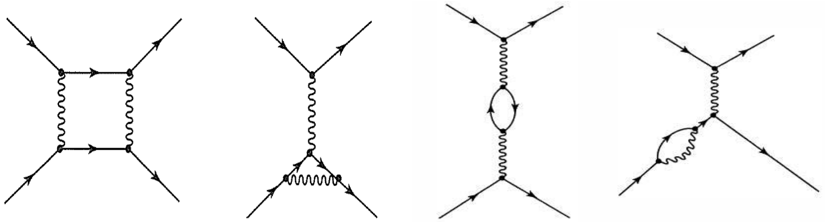
Loop diagrams:

Example: Higher order contributions to Møller scattering $e^- + e^- \to e^- + e^-$



All the above diagrams are of order $\alpha^2 \sim e^4$

Perturbative series in α $\to$ Radiative (Quantum) corrections

$$\text{Scattering amplitude } \mathcal{M} = \alpha \mathcal{M}_1 + \alpha^2 \mathcal{M}_2 + \alpha^3 \mathcal{M}_3 + \mathcal{O}(\alpha^4)$$

with the fine structure constant

$$\alpha = \frac{e^2}{4\pi}$$

Perturbative computation of $\mathcal{M}$

$$\text{Scattering amplitude } \mathcal{M} = \sum_{k=1}^{\infty} \alpha^k \mathcal{M}_k$$

- Perturbaive series is an infinite polynomial series in the orders of α
- Perturbative calcuation of $\mathcal{M}$ is only valid if $\alpha \sim e^2$ is small enough
- For QED, $\alpha = \frac{1}{137}$ and is small enough
- Because α is such a small number, diagrams with more and more vertices contribute less and less to the final result !!

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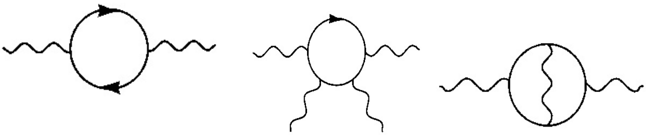
Question: How to compute $\mathcal{M}_k$'s?

- Tree level computation (Later in this lecture)
- Loop computation (in QFT lectures)

n-point correlation (Green's) functions of QED and loop corrections

$$
\begin{aligned}
V &= \text{Number of vertices} \\
E_e &= \text{Number of external lines for electrons} \\
I_e &= \text{Number of internal lines for electrons} \\
E_p &= \text{Number of external lines for photons} \\
I_p &= \text{Number of internal lines for photons}
\end{aligned}
$$

- n in ***n*-point function** is given by $n = E_e + E_p$
- L =loop order of a diagram is given by $L = (I_e + I_p) - (V - 1)$



$L = 2 - 1 = 1$ $\qquad$ $L = 4 - 3 = 1$ $\qquad$ $L = 5 - 3 = 2$

For each *n*-point function we have a separate perturbative series

$$\boxed{\mathcal{M}_n = \sum_{k=1}^{\infty} \alpha^k \mathcal{M}_{kn}}$$

Self-energy and vertex diagrams