Introduction to Elementary Particle Physics

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Elementary Particle Physics

Lecture 13: Farvardin 19, 1398

1397-98-II

Lecture 13

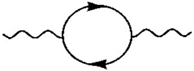
Radiative Corrections of QED

Lecture 13

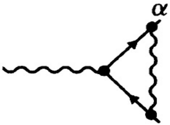
Remark 1: Important one-loop corrections to QED

- Fermion self-energy diagram (Order g^2)

- Vacuum polarization or photon self-energy diagram (Order g^2)

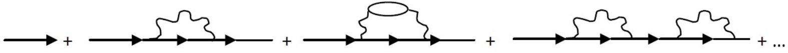


- Vertex function (Order $g \times g^2$)

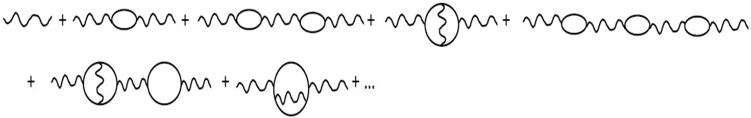


Remark 2: Perturbative series in QED

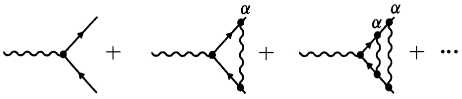
- Perturbative corrections to **fermion propagator** (including one-loop fermion self-energy)$\rightarrow$ **Effective (constituent) mass of fermions**



- Perturbative corrections to **photon propagator** (including one-loop vacuum polarization tensor)$\rightarrow$ **Running coupling of QED**

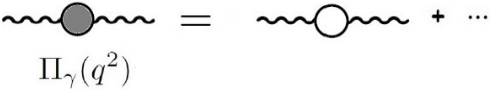


- Perturbative corrections to **vertex** (including one-loop vertex function) $\rightarrow$ **Anomalous magnetic moment of fermions**

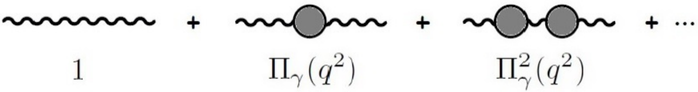


Remark 3: Photon propagator; Radiative corrections

Summation of all one-particle irreducible (1PI) diagrams



Summation of all orders



$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Pi_\gamma(q^2)}$$

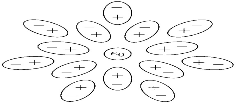
Taylor Expansion
[Geometric series]

Lecture 13

Remark 3: Screening of the electric charge (Running coupling)

- Classical Electrodynamics
  - Effective coupling depends on how far you are from the source
  - In a dielectric medium $q_{eff} = \frac{q}{\epsilon}$ with ϵ the dielectric constant
  - The closer we are to the positive charge, the more we see the full charge q

- Quantum Electrodynamics
  - Vacuum itself behaves like a dielectric medium $\rightarrow$ vacuum polarization



Uehling potential ($r \gg \frac{1}{m_e}$ with m_e the electron mass)

$$V(r) = -\frac{\alpha}{r}\left(1 + \frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2m_e r}}{(m_e r)^{3/2}} + \cdots\right)$$

Note: **Compton wavelength** $\lambda_c = \frac{\hbar}{m_e c} = 2.43 \times 10^{-12}$ m.
For $\hbar = c = 1$, we have $r \sim \frac{1}{m_e} = \lambda_c \sim 4 \times 10^{-3}$ A$^\circ$

- At $r < \frac{1}{m_e}$, we begin to penetrate the polarization cloud and see the **bare charge**

Remark 4: β-function of QED

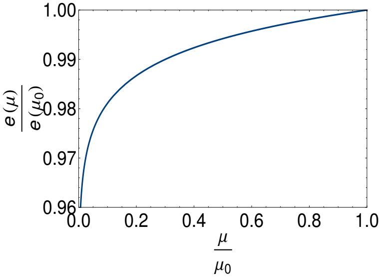
For $\hbar = c = 1$, Energy scale $= \mu = \frac{1}{\text{Length scale}} = \frac{1}{r}$

- **Definition: β-function**

$$\beta(e(\mu)) \equiv \mu \frac{\partial e}{\partial \mu}$$

- One-loop β-function of QED

$$\beta(e(\mu)) = \mu \frac{\partial e(\mu)}{\partial \mu} = \frac{e^3}{12\pi^2} \rightarrow e^2(\mu) = \frac{e^2(\mu_0)}{1 - \frac{e^2(\mu_0)}{6\pi^2} \ln \frac{\mu}{\mu_0}}$$



The four (three) forces

B. Quantum Chromodynamics (QCD)

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Remark 1: Primitive vertices of QCD

$$\mathcal{L}_1 = +g_s\bar{\psi}\gamma^\mu A_\mu \psi$$

$$\mathcal{L}_2 = -g_s(\partial_\mu A_\lambda)[A^\mu, A^\lambda]$$

$$\mathcal{L}_3 = -g_s^2[A_\mu, A_\nu][A^\mu, A^\nu]$$

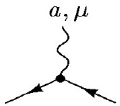
$$\mathcal{L}_4 = -g_s\bar{c}[\partial^\mu A_\mu, c]$$

with $A_\mu = A_\mu^a t^a$

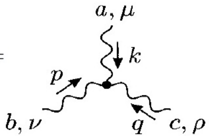
$t^a, a = 1, \cdots, 8$ are Gell-Mann matrices

Gell-Mann matrices are generators of $SU(3)$ gauge group

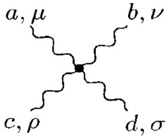
$\mathcal{L}_1 =$



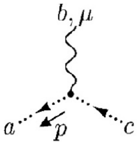
$\mathcal{L}_2 =$



$\mathcal{L}_3 =$



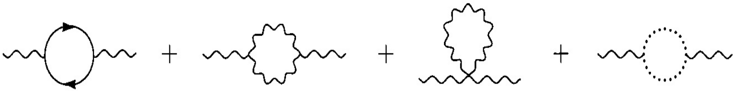
$\mathcal{L}_4 =$



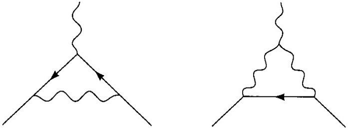
Remark 2: Important one-loop corrections to QCD

- Fermion (quark) self-energy diagram (Order g_s^2)

- Vacuum polarization or gluon self-energy diagram (Order g_s^2)



- Vertex function (Order g_s^2)



$\rightarrow$ **Running coupling of QCD**

Remark 3: Antiscreening, β-function of QCD [1974] (Nobel prize 2004)

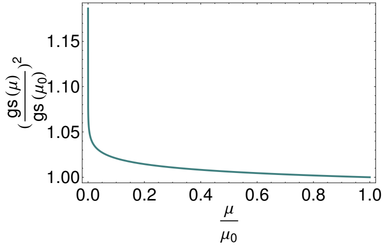
- One-loop β-function of QCD

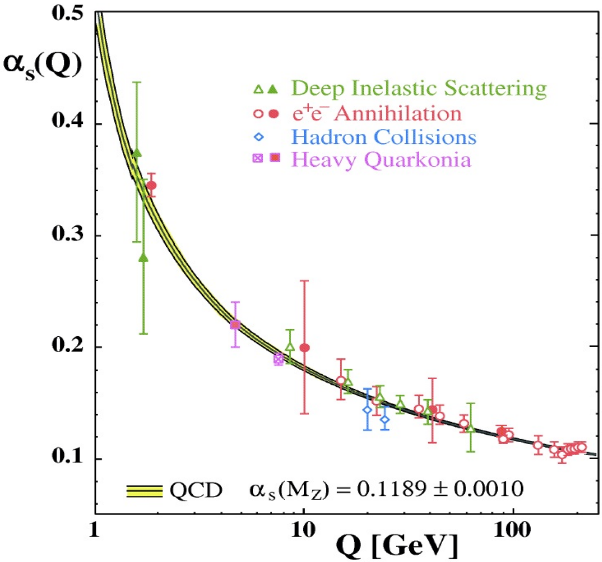
$$\beta(g_s(\mu)) \equiv \mu \frac{\partial g_s}{\partial \mu}$$

- One-loop β-function of QED

$$\beta(g_s(\mu)) = \mu \frac{\partial g_s(\mu)}{\partial \mu} = -\frac{g_s^3}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right) \Longrightarrow$$

$$g_s^2(\mu) = \frac{g_s^2(\mu_0)}{1 + \frac{g_s^2(\mu_0)}{8\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\ln\frac{\mu}{\mu_0}}$$





Summary α_s
[Bethke 2006]