Lecture 10-QM_II (01/08/00)

Saturday, October 23, 2021 3:00 PM

$$\vec{J}_1 + \vec{J}_2 = \vec{J}$$

$$\{J_1^2, J_{1z}, J_2^2, J_{2z}\} \rightarrow \{J^2, J_z; J_1^2, J_2^2\}$$

$$[J_{1i}, J_{1j}] = i\epsilon_{ijk} J_{1k}$$

$$[\vec{J}_1, \vec{J}_2] = 0$$

$$|j_1, m_{j_1}\rangle \otimes |j_2, m_{j_2}\rangle \rightarrow |j, m_j\rangle$$

- $j_1 = s_1 = \frac{1}{2}$ $\qquad j_2 = s_2 = \frac{1}{2}$

$$\mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} = \mathcal{H}_0 + \mathcal{H}_1$$

$$(2j_1+1) \qquad 2 \otimes 2 = 1 + 3$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow\rangle$$

$$|1,-1\rangle = |\downarrow\downarrow\rangle$$

$$|1,0\rangle \ \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$$

$$|1,1\rangle = |\uparrow\uparrow\rangle$$

- $j_1 = l$ $\qquad j_2 = \frac{1}{2}$

$$\mathcal{H}_l \otimes \mathcal{H}_{1/2} = \mathcal{H}_{l-\frac{1}{2}} + \mathcal{H}_{l+\frac{1}{2}}$$

$$(2l+1)2 = \left(2(l-\tfrac{1}{2})+1\right) + \left(2(l+\tfrac{1}{2})+1\right) = 2(2l+1)$$

ادعا: $\quad |j_1 - j_2| \leq j \leq j_1 + j_2$

$$-j \leq m_j \leq j \qquad m_j = m_{j_1} + m_{j_2}$$

$$\boxed{\begin{gathered}\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} = \mathcal{H}_{|j_1-j_2|} \oplus \cdots\cdots \oplus \mathcal{H}_{j_1+j_2} \\ (2j_1+1)(2j_2+1) = \sum_{j=|j_1-j_2|}^{j_1+j_2} (2j+1) = (2j_1+1)(2j_2+1)\end{gathered}}$$

$J_z = J_{1z} + J_{2z} \qquad m_{j_1} + m_{j_2} = m_j$. الف) اثبات

$$J_z |j, m_j\rangle = \hbar m_j |j, m_j\rangle$$

$$\langle j_1 m_{j_1}; j_2 m_{j_2} | \underbrace{J_z}_{J_{1z}+J_{2z}} | j, m_j\rangle = \hbar m_j \langle j_1, m_{j_1}; j_2, m_{j_2} | j, m_j\rangle$$

طرف چپ رابطه

$$(\hbar m_{j_1} + \hbar m_{j_2}) \langle j_1 m_{j_1}; j_2 m_{j_2} | j\, m_j\rangle = \hbar m_j \langle j_1 m_{j_1}, j_2 m_{j_2} | j\, m_j\rangle$$

$$m_{j_1} + m_{j_2} - m_j = 0 \rightarrow \boxed{m_{j_1} + m_{j_2} = m_j}$$

$|j_1 - j_2| \leq j \leq j_1 + j_2$ ب) اثبات

$$\left.\begin{aligned} m_j &= m_{j_1} + m_{j_2} \\ -j_1 &\leq m_{j_1} \leq j_1 \\ -j_2 &\leq m_{j_2} \leq j_2 \end{aligned}\right\} \qquad \begin{aligned} (m_j)_{max} &= j_1 + j_2 \\ (m_j)_{min} &= -(j_1+j_2) \end{aligned}$$

$$-(j_1+j_2) \leq m_j \leq j_1 + j_2$$

$$j_1 = 3 \qquad j_2 = 2 \qquad -5 \leq m_j \leq 5$$

$$-3 \leq m_{j_1} \leq 3 \qquad -2 \leq m_{j_2} \leq 2$$

m_j	(m_{j_1}, m_{j_2})		تبهگنی در m_j
5	$(3, 2)$		1
4	$(3, 1)$ $(2, 2)$	$\nwarrow \nearrow m_{j_2}$	2

5		(3, 2)					1
4		(3, 1)	(2, 2)				2
3		(3, 0)	(2, 1)	(1, 2)	~~(0, 3)~~ $\to m_{j_2}$		3
2		(3, -1)	(2, 0)	(1, 1)	(0, 2)		4
1		(3, -2)	(2, -1)	(1, 0)	(0, 1)	(-1, 2)	5
0	~~(-3, 3)~~	(2, -2)	(1, -1)	(0, 0)	(-1, 1)	(-2, 2)	5
-1		(1, -2)	(0, -1)	(-1, 0)	(-2, 1)	(-3, 2)	5
-2		(0, -2)	(-1, -1)	(-2, 0)	(-3, 1)		4
-3		(-3, 0)	(-2, -1)	(-1, -2)			3
-4		(-3, -1)	(-2, -2)				2
-5		(-3, -2)					1
							35

لطفاً به جدول آخر صفحه توجه نمایید

$$(2j_1+1)\ (2j_2+1)$$
$$(2\times 3+1)\ (2\times 2+1)$$
$$7 \times 5 = 35$$

$$|j_1 - j_2| \leq j \leq j_1 + j_2 \quad \checkmark$$ ادعا

$$1 \leq j \leq 5$$ مثلاً در مورد فوق

$j = 5 \qquad m_j = \pm 5, \pm 4, \pm 3, \pm 2, \pm 1, 0$

$j = 4 \qquad m_j = \pm 4, \pm 3, \pm 2, \pm 1, 0$

$j = 3 \qquad m_j = \pm 3, \pm 2, \pm 1, 0$

$j = 2 \qquad m_j = \pm 2, \pm 1, 0$

$j = 1 \qquad m_j = \pm 1, 0$

$m_j \to$	5	4	3	2	1	0	-1	-2	-3	-4	-5
m_j تعداد تبهگنی	1	2	3	4	5	5	5	4	3	2	1

جمع تبهگنی $= 35$

$(2j+1)$

$$\mathcal{H}_3 \otimes \mathcal{H}_2 = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4 + \mathcal{H}_5$$
$$7 \times 5 = 3 + 5 + 7 + 9 + 11$$
$$35 = 35 \ \checkmark$$

$$\boxed{\begin{array}{l} \mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} = \mathcal{H}_{|j_1-j_2|} + \cdots \mathcal{H}_{j_1+j_2} \\ (2j_1+1)(2j_2+1) = \sum_{j=|j_1-j_2|}^{j_1+j_2} (2j+1) = (2j_1+1)(2j_2+1) \end{array}}$$

- ضرایب Clebsch - Gordan

$$|j\ m_j\rangle = \sum_{m_{j_1}} C_{j_1 m_{j_1},\, j_2 m_{j_2}} |j_1\ m_{j_1}\rangle \otimes |j_2\ m_{j_2}\rangle$$

$$m_{j_2} = m_j - m_{j_1}$$

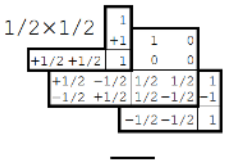
$$|1, 0\rangle = \frac{1}{\sqrt{2}} (\uparrow\downarrow + \downarrow\uparrow)$$

		j_1' m_{j_1}	j_2' m_{j_2}
	$j_1 \otimes j_2$		
m_1	m_2	a	$-b$
m_1'	m_2'	c	d

$$\left.\begin{array}{l} a + c = 1 \\ b + d = 1 \\ a + b = 1 \\ c + d = 1 \end{array}\right\} \to \begin{array}{l} b = c \\ a = d \end{array}$$

$$\begin{cases} |j_1'\ m_{j_1}\rangle = \sqrt{a}\ \underbrace{|j_1\ m_1\rangle \otimes |j_2\ m_2\rangle} + \sqrt{c}\ \underbrace{|j_1\ m_1'\rangle \otimes |j_2\ m_2'\rangle} \\ |j_2'\ m_{j_2}\rangle = -\sqrt{b}\ |j_1\ m_1\rangle \otimes |j_2\ m_2\rangle + \sqrt{d}\ |j_1\ m_1'\rangle \otimes |j_2\ m_2'\rangle \end{cases}$$

$$\begin{cases} |j_1\ m_1\rangle \otimes |j_2\ m_2\rangle = \sqrt{a}\ |j_1'\ m_{j_1}\rangle - \sqrt{b}\ |j_2'\ m_{j_2}\rangle \\ |j_1\ m_1'\rangle \otimes |j_2\ m_2'\rangle = \sqrt{c}\ |j_1'\ m_{j_1}\rangle + \sqrt{d}\ |j_2'\ m_{j_2}\rangle \end{cases}$$



1/2×1/2

1 +1 | 1 0 | 0 0
+1/2 +1/2 | 1
+1/2 −1/2 | 1/2 1/2 | 1
−1/2 +1/2 | 1/2 −1/2 | −1
−1/2 −1/2 | 1

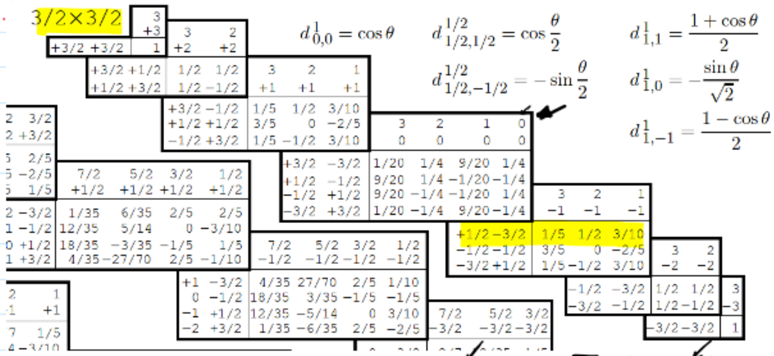
$$\overset{\uparrow}{|\tfrac{1}{2},\tfrac{1}{2}\rangle} \otimes \overset{\uparrow}{|\tfrac{1}{2},\tfrac{1}{2}\rangle} = |1,1\rangle$$

s, m_s

$$\overset{\uparrow}{|\tfrac{1}{2}\ \tfrac{1}{2}\rangle} \otimes \overset{\downarrow}{|\tfrac{1}{2},-\tfrac{1}{2}\rangle} = \sqrt{\tfrac{1}{2}}\ |1,0\rangle + \sqrt{\tfrac{1}{2}}\ |0,0\rangle$$

$$\overset{\downarrow}{|\tfrac{1}{2},-\tfrac{1}{2}\rangle} \otimes \overset{\uparrow}{|\tfrac{1}{2},\tfrac{1}{2}\rangle} = \sqrt{\tfrac{1}{2}}\ |1,0\rangle - \sqrt{\tfrac{1}{2}}\ |0,0\rangle$$

$$\overset{\downarrow}{|\tfrac{1}{2},-\tfrac{1}{2}\rangle} \otimes \overset{\downarrow}{|\tfrac{1}{2},-\tfrac{1}{2}\rangle} = |1,-1\rangle$$



3/2×3/2

3 +3 | 3 2 +2 +2
+3/2 +3/2 | 1
+3/2 +1/2 | 1/2 1/2 | 3 2 1 +1 +1 +1
+1/2 +3/2 | 1/2 −1/2
+3/2 −1/2 | 1/5 1/2 3/10 | 3 2 1 0 0 0 0 0
+1/2 +1/2 | 3/5 0 −2/5
−1/2 +3/2 | 1/5 −1/2 3/10
+3/2 −3/2 | 1/20 1/4 9/20 1/4
+1/2 −1/2 | 9/20 1/4 −1/20 −1/4 | 3 2 1 −1 −1 −1
−1/2 +1/2 | 9/20 −1/4 −1/20 1/4
−3/2 +3/2 | 1/20 −1/4 9/20 −1/4
+1/2 −3/2 | 1/5 1/2 3/10
−1/2 −1/2 | 3/5 0 −2/5 | 3 2 −2 −2
−3/2 +1/2 | 1/5 −1/2 3/10
−1/2 −3/2 | 1/2 1/2 | 3 −3
−3/2 −1/2 | 1/2 −1/2
−3/2 −3/2 | 1

2 3/2
2 +3/2
5 2/5
5 −2/5 | 7/2 5/2 3/2 1/2 +1/2 +1/2 +1/2 +1/2
5 1/5
2 −3/2 | 1/35 6/35 2/5 2/5
1 −1/2 | 12/35 5/14 0 −3/10
0 +1/2 | 18/35 −3/35 −1/5 1/5 | 7/2 5/2 3/2 1/2
1 +3/2 | 4/35 −27/70 2/5 −1/10 | −1/2 −1/2 −1/2 −1/2
+1 −3/2 | 4/35 27/70 2/5 1/10
0 −1/2 | 18/35 3/35 −1/5 −1/5
−1 +1/2 | 12/35 −5/14 0 3/10 | 7/2 5/2 3/2
−2 +3/2 | 1/35 −6/35 2/5 −2/5 | −3/2 −3/2 −3/2
2 1
1 +1
7 1/5
4 −3/10

$$d^{1}_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1}_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^{1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^{1}_{1,-1} = \frac{1-\cos\theta}{2}$$

mj	mj [illegible]
0	4
1	3
2	2
3	1
−1	3
−2	2
−3	1
	16

$$\underset{j_1}{|\tfrac{3}{2},\tfrac{1}{2}\rangle} \otimes |\tfrac{3}{2},-\tfrac{3}{2}\rangle = \sqrt{\tfrac{1}{5}}\ |\underset{j}{3},-1\rangle + \sqrt{\tfrac{1}{2}}\ |\underset{j}{2},-1\rangle + \sqrt{\tfrac{3}{10}}\ |\underset{j}{1},-1\rangle$$

$m_{j_1} = \frac{1}{2}$ $\qquad m_{j_1} + m_{j_2} = -1 = m_j$

$m_{j_2} = -\frac{3}{2}$

$j_1 = \frac{3}{2}$ $\qquad j_1 - j_2 \leqslant j \leqslant j_1 + j_2$

$j_2 = \frac{3}{2}$ $\qquad 0 \leqslant j \leqslant 3$

$j = 0, 1, 2, 3$

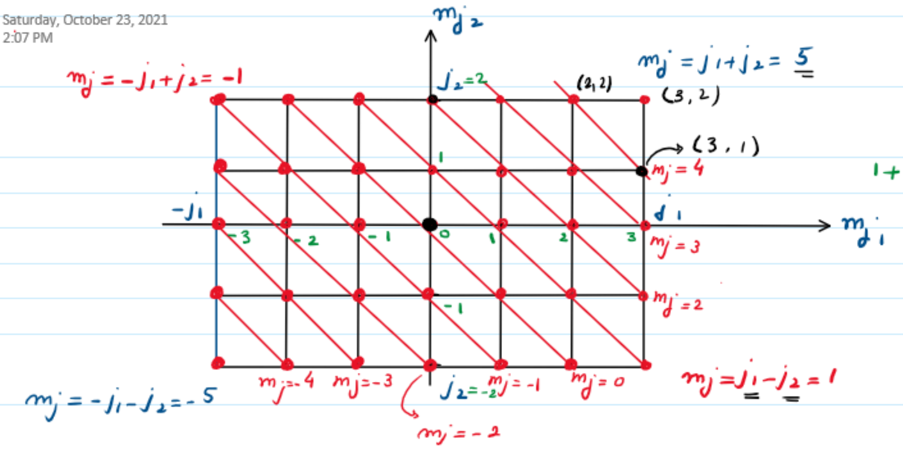
$$\mathcal{H}_{3/2} \otimes \mathcal{H}_{3/2} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3$$

$$4 \times 4 = 1 + 3 + 5 + 7$$

$$16 = 16$$

Grifiths – Introduction to elementary particle physics

Saturday, October 23, 2021
2:07 PM



$j_1 = 3$

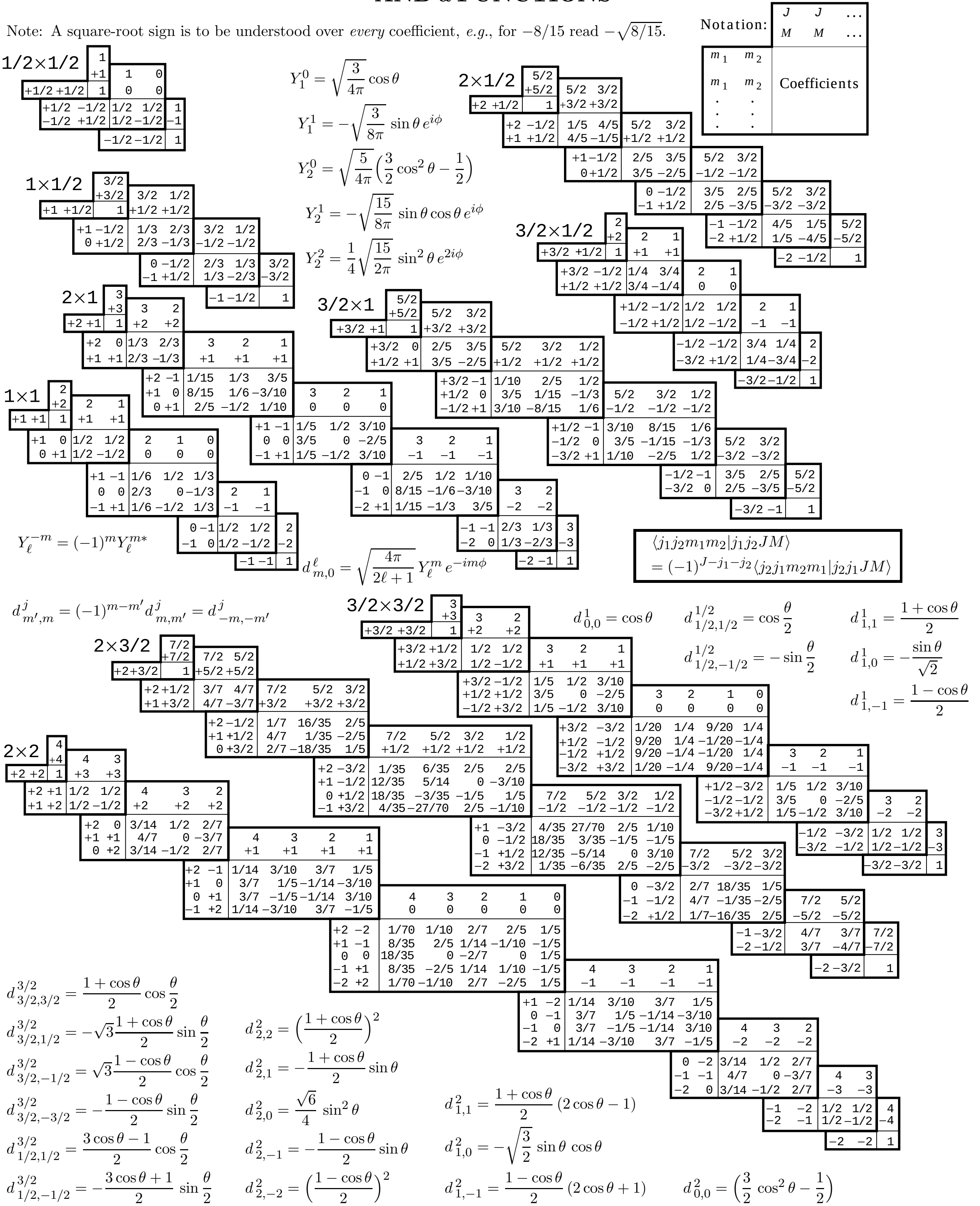
$j_2 = 2$

$1 + 2 + 3 + 4 + 5 + 5 + 5 + 4 + 3 + 2 + 1$

$= 35$

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.



Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			
.	.			
.	.			

1/2×1/2

		1
		+1
+1/2	+1/2	1

		1	0
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

		1
		−1
−1/2	−1/2	1

1×1/2

		3/2
		+3/2
+1	+1/2	1

		3/2	1/2
		+1/2	+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

		3/2	1/2
		−1/2	−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

		3/2
		−3/2
−1	−1/2	1

2×1/2

		5/2
		+5/2
+2	+1/2	1

		5/2	3/2
		+3/2	+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

		5/2	3/2
		+1/2	+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

		5/2	3/2
		−1/2	−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

		5/2	3/2
		−3/2	−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

		5/2
		−5/2
−2	−1/2	1

3/2×1/2

		2
		+2
+3/2	+1/2	1

		2	1
		+1	+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

		2	1
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

		2	1
		−1	−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

		2
		−2
−3/2	−1/2	1

2×1

		3
		+3
+2	+1	1

		3	2
		+2	+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

		3	2	1
		+1	+1	+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

		3	2	1
		0	0	0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

		3	2	1
		−1	−1	−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

		3	2
		−2	−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

		3
		−3
−2	−1	1

1×1

		2
		+2
+1	+1	1

		2	1
		+1	+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

		2	1	0
		0	0	0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

		2	1
		−1	−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

		2
		−2
−1	−1	1

3/2×1

		5/2
		+5/2
+3/2	+1	1

		5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

		5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

		5/2	3/2	1/2
		−1/2	−1/2	−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

		5/2	3/2
		−3/2	−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

		5/2
		−5/2
−3/2	−1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\,\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

3/2×3/2

		3
		+3
+3/2	+3/2	1

		3	2
		+2	+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

		3	2	1
		+1	+1	+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

		3	2	1	0
		0	0	0	0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

		3	2	1
		−1	−1	−1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

		3	2
		−2	−2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

		3
		−3
−3/2	−3/2	1

$$d^{\,1}_{0,0} = \cos\theta \qquad d^{\,1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{\,1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{\,1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{\,1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{\,1}_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

		7/2
		+7/2
+2	+3/2	1

		7/2	5/2
		+5/2	+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

		7/2	5/2	3/2
		+3/2	+3/2	+3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

		7/2	5/2	3/2	1/2
		+1/2	+1/2	+1/2	+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

		7/2	5/2	3/2	1/2
		−1/2	−1/2	−1/2	−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

		7/2	5/2	3/2
		−3/2	−3/2	−3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

		7/2	5/2
		−5/2	−5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

		7/2
		−7/2
−2	−3/2	1

2×2

		4
		+4
+2	+2	1

		4	3
		+3	+3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

		4	3	2
		+2	+2	+2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

		4	3	2	1
		+1	+1	+1	+1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

		4	3	2	1	0
		0	0	0	0	0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

		4	3	2	1
		−1	−1	−1	−1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

		4	3	2
		−2	−2	−2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

		4	3
		−3	−3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

		4
		−4
−2	−2	1

$$d^{\,3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^{\,2}_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{\,3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^{\,2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{\,3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^{\,2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^{\,2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{\,3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2} \qquad d^{\,2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^{\,2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\,\cos\theta$$

$$d^{\,3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2} \qquad d^{\,2}_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2 \qquad d^{\,2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^{\,2}_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 34.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.