Lecture 15-QM_II (17/08/00)

Monday, November 8, 2021 2:51 PM

اتم هیدروژن واقعی

۱) تصحیح طیف اتم هیدروژن به دلیل تصحیح نسبیتی انرژی جنبشی

$$H_0 = \frac{\vec{P}^2}{2m_e} - \frac{Ze^2}{r}$$

$$E = \sqrt{\vec{P}^2c^2 + m^2c^4} \sim \frac{P^2}{2m_e} + m_e c^2$$

$$H_1 = -\frac{1}{8}\frac{(\vec{P}^2)^2}{m_e^3 c^2} = \frac{-1}{2m_e c^2}\left(H_0 + \frac{Ze^2}{r}\right)^2$$

$$\frac{P^2}{m^2c^2} \ll 1 \qquad v^2 \ll c^2$$

$$E - m_e c^2 = E_{kin}$$

$$H_0 \underbrace{|n\rangle^{(0)}}_{|nlm_l\rangle^{(0)}} = E_n^{(0)} \underbrace{|n^{(0)}\rangle}_{|nlm_l\rangle^{(0)}}$$

$$E_n^{(0)} = -\frac{1 R_y}{n^2} \qquad R_y = \frac{1}{2} m_e c^2 (Z\alpha)^2$$

→ درجه تبهگنی $2n^2$

$n \qquad l = 0, \dots, n-1 \qquad -l \le m_l \le l \quad (2l+1)$

$$2n^2 = 2\sum_{l=0}^{n-1}(2l+1)$$

$$[H_0, H_1] = 0$$

$$|n\, l\, m_l\rangle^{(0)}$$

H_1 قطری است $\qquad E_n^{(1)} = {}^{(0)}\langle nlm_l | H_1 | nlm_l\rangle^{(0)}$

$$E_n^{(1)} = -\frac{1}{2}\frac{m_e c^2 (Z\alpha)^4}{n^4}\left(\frac{n}{l+\frac{1}{2}} - \frac{3}{4}\right)$$

اثر اسپین - مدار

یادآوری :

$$H\psi(\vec{x}) = E\psi(x)$$

$$H = \frac{1}{2m}\left(\vec{P} - \frac{q}{c}\vec{A}\right)^2 + q\varphi + V(\vec{x})$$

$$\vec{A} = -\frac{1}{2}(y, -x, 0) \qquad \varphi = 0$$

$$H_{magn} = -\frac{q}{2mc}\vec{L}\cdot\vec{B} \underset{\substack{q=-e \\ m=m_e}}{=} \mu_B \frac{\vec{L}}{\hbar}\cdot\vec{B}$$

$$\mu_B = \frac{e\hbar}{2m_e c}$$

$$H_{magn} = \mu_B \frac{1}{\hbar}(\vec{L} + 2\vec{S})\cdot\vec{B}$$

Paschen - Back

$g_s = 2$ ضریب ژیرومغناطیسی

$$\vec{B} = -\frac{q}{c}\vec{v}\times\vec{E} \qquad q = +1 \qquad \vec{v} = \frac{\vec{P}}{m_e}$$

میدان الکتریکی

$$\vec{E} = -\vec{\nabla}\varphi - \frac{1}{c}\frac{\partial}{\partial t}\vec{A} = -\vec{\nabla}\varphi$$

$$\vec{A} = \vec{0}$$

$$\vec{E} = -\vec{\nabla}\left(+\frac{Ze}{r}\right) = -\frac{\vec{r}}{r}\frac{\partial}{\partial r}\frac{1}{r}Ze = +\frac{Ze}{r^3}\vec{r}$$

$$\vec{B} \sim \vec{L}$$

$$\vec{S}\cdot\vec{B} \sim \vec{S}\cdot\vec{L}$$

$$\vec{B} = -\frac{1}{c}\frac{\vec{P}}{m_e}\times\left(+\frac{Ze}{r^3}\vec{r}\right)$$

$$= \frac{Ze}{m_e c\, r^3}(\vec{r}\times\vec{P}) = \frac{Ze}{m_e c\, r^3}\vec{L}$$

$$-\vec{\mu}_S\cdot\vec{B}$$

$$H_{magn}^{\text{اسپین}} = \frac{\mu_B}{\hbar} g_s \vec{S}\cdot\vec{B}$$

$$= +\frac{e^2 g_s}{2m_e^2 c^2}\frac{Z}{r^3}\vec{S}\cdot\vec{L} = \frac{e^2 Z g_s}{2m_e^2 c^2 r^3}\vec{S}\cdot\vec{L} = H_2$$

تصحیح اسپین - مدار

$$H = H_0 + H_2 \qquad E_n^{(1)} = \langle H_r \rangle$$

$$|n\,\ell\, m_\ell\rangle \otimes |s, m_s\rangle$$

$$\vec{J} = \vec{S} + \vec{L} \qquad \vec{J}^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{S}.\vec{L} \qquad [\vec{S}, \vec{L}]$$

$$\boxed{\vec{S}.\vec{L} = \vec{J}^2 - \vec{L}^2 - \vec{S}^2}$$

$$\langle \text{پایه مناسب} | \vec{S}.\vec{L} | \text{پایه مناسب} \rangle = \alpha \underbrace{\langle \text{پایه مناسب} | \text{پایه مناسب} \rangle}_{1}$$

$$\{\underset{n}{H_0}, \underset{\ell}{\vec{L}^2}, \underset{m_\ell}{L_z}, \underset{s}{\vec{S}^2}, \underset{m_s}{S_z}\} \rightarrow \{\underset{n}{H_0}, \underset{j}{\vec{J}^2}, \underset{m_j}{J_z}, \underset{\ell}{\vec{L}^2}, \underset{s=\frac{1}{2}}{\vec{S}^2}\}$$

$$|n\,\ell\, m_\ell\rangle |s, m_s\rangle \rightarrow |n\, j\, m_j\, \ell\, s\rangle$$

$$\vec{S}.\vec{L}\, |j\, m_j\rangle = \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)\, |j\, m_j, \ell, s\rangle$$

$$= \frac{1}{2}\hbar^2 \underbrace{\left(j(j+1) - \ell(\ell+1) - s(s+1)\right)}_{\alpha} \underbrace{|j\, m_j, \ell, s\rangle}_{|\text{پایه مناسب}\rangle}$$

$$\vec{J} = \vec{L} + \vec{S} \qquad s = \frac{1}{2} \qquad m_s = \pm\frac{1}{2}$$

$$j = \ell - \frac{1}{2},\ \ell + \frac{1}{2} \qquad m_j = m_\ell + m_s = m_\ell \pm \frac{1}{2}$$

$$|j, m_j\rangle = \alpha_\pm\, |\ell, m_\ell = m_j - \tfrac{1}{2}\rangle |\uparrow\rangle + \beta_\pm\, |\ell, m_\ell = m_j + \tfrac{1}{2}\rangle |\downarrow\rangle$$

α :

$$j = \ell + \frac{1}{2}, \quad s = \frac{1}{2} \qquad \alpha = \ell$$

$$j = \ell - \frac{1}{2}, \quad s = \frac{1}{2} \qquad \alpha = -\ell - 1$$

$$\overset{\rightarrow(0)}{\langle} n, j, m_j, \ell, s = \tfrac{1}{2} | H_2 | n, j, m_j, \ell, s = \tfrac{1}{2} \rangle^{(0)\leftarrow} = E_n^{(1)}$$

$$E_n^{(1)} = {}^{(0)}\langle n, j, m_j | \frac{Ze^2 g_s}{2m_e^2 c^2\, r^3}\, \underline{\vec{S}.\vec{L}}\, | n\, j\, m_j \rangle^{(0)}$$

$$E_n^{(1)} = \frac{Ze^2 g_s}{2m_e^2 c^2} \underbrace{\begin{pmatrix} \ell : j = \ell + \frac{1}{2} \text{ برای} \\ -\ell - 1 : j = \ell - \frac{1}{2} \text{ 〃} \end{pmatrix}}_{\text{بخش اسپینی}} \underbrace{{}^{(0)}\langle n\, j\, m_j, \ell, s | \frac{1}{r^3} | n\, j\, m_j\, \ell, s \rangle^{(0)}}_{\text{بخش مکانی}}$$

$$\langle \vec{r} | n\, j\, m_j \rangle = \alpha_\pm \underbrace{\langle \vec{r} | n\, \ell\, m_\ell \rangle^{(0)}}_{R_{n\ell}(r)\, Y_{\ell m}(\theta, \varphi)} |\uparrow\rangle + \beta_\pm \langle \vec{r} | n\, \ell\, m_\ell \rangle^{(0)} |\downarrow\rangle$$

بخش مکانی

$$\langle R_{n\ell}(r) | \frac{1}{r^3} | R_{n\ell}(r) \rangle = \frac{Z^3}{a_B^3\, \ell(\ell+1)(\ell+\frac{1}{2})\, n^3} \qquad \ell \geq 1$$

$$a_B = \frac{\hbar}{m_e c \alpha}$$

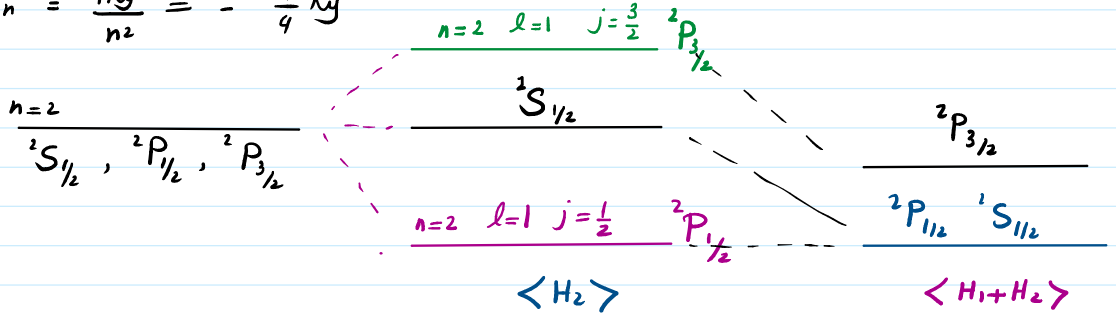
$$\boxed{E_n^{(1)} = \langle H_2 \rangle = \frac{m_e c^2 (Z\alpha)^4}{4n^3\, \ell(\ell+1)\, (\ell + \frac{1}{2})} \begin{cases} \ell \leftarrow & j = \ell + \frac{1}{2} \\ -\ell - 1 & j = \ell - \frac{1}{2} \end{cases}} \qquad \ell \geq 1$$

$n = 2 \qquad \ell = 0, 1$

$\ell = 0 \qquad s = \frac{1}{2} \qquad j = \frac{1}{2} \qquad {}^{n=2}S_{1/2 = j} \qquad {}^2S_{1/2}$

$\ell = 1 \qquad s = \frac{1}{2} \rightarrow j = \frac{1}{2}, \frac{3}{2} \qquad {}^2P_{1/2 = j} \qquad {}^2P_{3/2 = j}$

$$E_n^{(0)} = \frac{-1 R_y}{n^2} = -\frac{1}{4} R_y$$



$$E_n^{(1)} = \langle H_1 + H_2 \rangle = \frac{m_e c^2 (Z\alpha)^4}{2n^4}\left(\frac{3}{4} - \frac{n}{j+\frac{1}{2}}\right)$$

$j = \frac{1}{2} \;,\; \frac{3}{2}$ $\qquad \underbrace{{}^2S_{1/2} \quad {}^2P_{1/2}}_{j=\frac{1}{2}} \qquad \underset{j=\frac{3}{2}}{{}^2P_{3/2}}$

$l=0$

$$V(\vec{x}) = -\frac{Ze^2}{r}$$

$$V(\vec{x}+\delta\vec{x}) = V(\vec{x}) + \delta\vec{x}\cdot\vec{\nabla}V(\vec{x}) + \frac{1}{2}\sum_{i,j}\delta x_i\,\delta x_j\,\frac{\partial^2 V}{\partial x_i \partial x_j} + \cdots$$

$$\langle V(\vec{x}+\delta\vec{x})\rangle = V(\vec{x}) + \cancel{\langle\delta\vec{x}\rangle\cdot\vec{\nabla}V}$$

$$+ \frac{1}{2}\sum_{i,j}\langle\delta x_i\,\delta x_j\rangle\frac{\partial^2 V}{\partial x_i\partial x_j} + \cdots$$

$$\langle\delta\vec{x}\rangle = 0 \qquad \langle\delta x_i\,\delta x_j\rangle = \langle\delta x_i^2\rangle\,\delta_{ij} \quad \text{بدون جمع } i$$

$$= \frac{1}{3}\langle\delta\vec{x}^2\rangle\,\delta_{ij}$$

$$\langle V(\vec{x}+\delta\vec{x})\rangle = V(\vec{x}) + \frac{1}{6}\sum_{\cancel{i,j}}\langle\delta\vec{x}^2\rangle\frac{\partial^2 V}{\partial x_i\,\partial x_{j=i}}\cancel{\delta_{ij}} + \cdots$$

$H = \frac{p^2}{2m} + V(\vec{x})$

$$= V(\vec{x}) + \frac{1}{6}\langle\delta\vec{x}^2\rangle\,\underbrace{\Delta V(\vec{x})}_{\sum_i \frac{\partial^2 V}{\partial x_i^2}}$$

انحراف معیار $\sim \lambda_c^2$ طول موج کامپتون

$$= \left(\frac{\hbar}{m_e c}\right)^2$$

$l=0$

$$= V(\vec{x}) + \frac{1}{6}\,\frac{\hbar^2}{m_e^2 c^2}\,\Delta V(\vec{x}) + \cdots$$

$\frac{1}{8}$ $\qquad$ Zitter bewegung

لرزش حرکت

$$V(\vec{x}) = \frac{-Ze^2}{r} \qquad \Delta\left(-\frac{Ze}{r}\right) = 4\pi Z e^2\,\delta(\vec{x})$$

$$H_3 = \frac{\pi}{2}\,\frac{Ze^2\hbar^2}{m_e^2c^2}\,\delta(\vec{x})$$

$$E_n^{(1)} = {}^{(0)}\langle H_3\rangle^{(0)} = \frac{\pi}{2}\,\frac{Ze^2\hbar^2}{m_e^2c^2}\int d^3x\,|\psi_{nlm_l}(\vec{x})|^2\,\delta(\vec{x})$$

$$= \frac{\pi}{2}\,\frac{Ze^2\hbar^2}{m_e^2c^2}\,|\psi_{nlm_l}(\vec{0})|^2$$

$$= \frac{\pi}{2}\,\frac{Ze^2\hbar^2}{m_e^2c^2}\,\frac{Z^3}{\pi a_B^3 n^3}\,\delta_{l0}\,\delta_{m_l 0}$$

$$= \frac{(Z\alpha)^4}{2n^3}\,m_e c^2\,\delta_{l0}\,\delta_{m_l 0} \qquad l=0$$

$$\langle H_2\rangle_{j=l+\frac{1}{2}} = \frac{(Z\alpha)^4\;m_e c^2}{n^3\;4\cancel{l}(l+1)(l+\frac{1}{2})} \qquad \{\cancel{l} = \frac{(Z\alpha)^4 m_e c^2}{2n^3}$$

$$\langle H_2 \rangle_{j=\ell+\frac{1}{2}} = \frac{(Z\alpha)^4 \quad m_e c^2}{n^3 \quad 4\cancel{\ell}(\ell+1)(\ell+\frac{1}{2})} \quad \left\{ \cancel{\ell} = \frac{(Z\alpha)^4 m_e c^2}{2n^3} \right)$$

$$\ell \uparrow \quad \ell+1 \overset{!!}{=} 0 \quad \ell+\tfrac{1}{2} \overset{!!}{=} 0$$

$$(Z\alpha)^4 \sim (10^{-2})^4 \doteq 10^{-8}$$