Lecture 17-QM_II (24/08/00)

Monday, November 15, 2021 2:30 PM

اسپینی مغنا

$$\vec{\nabla}\times\vec{A} = \vec{B}_N$$

$$H_4 = -\vec{\mu}_s \cdot \vec{B}_N = -\gamma_e\gamma_N \left(\frac{1}{4\pi} S_i I_j \partial_i\partial_j \frac{1}{r} - \vec{S}\cdot\vec{I}\,\vec{\nabla}^2\frac{1}{r}\right)$$

$$\vec{B}_N = \gamma_N \frac{1}{4\pi}\left\{(\vec{I}\cdot\vec{\nabla})\vec{\nabla}\frac{1}{r} - \vec{I}\,\nabla^2\frac{1}{r}\right\}$$

$$\vec{\mu}_N = \gamma_N \vec{I} \qquad \gamma_N = \frac{Zeg_N}{2m_pc} \qquad \gamma_s = \frac{-eg_s}{2m_ec} \qquad \vec{\mu}_s = -\gamma_e\vec{S}$$

$$E_n^{(1)} = \langle H_4 \rangle$$

$$\langle H_4\rangle \overset{l=0}{=} -\frac{2}{3}\gamma_e\gamma_N |R_{n0}(0)|^2 \langle \vec{S}\cdot\vec{I}\rangle$$

| حرک مغنا > | حرک اسپینی >

$\langle \vec{L}\cdot\vec{S}\rangle$

$$\{H_0, \vec{L}^2, L_3, \vec{S}^2, S_3\} \quad |n\,l\,m_l\rangle|s,m_s\rangle \;\longrightarrow\; \{H_0, \vec{J}^2, J_3, L^2, S^2\}$$

$$\vec{J} = \vec{L}+\vec{S}$$

$$|j\ m_j\rangle$$

$$j = l-\tfrac{1}{2},\ l+\tfrac{1}{2}$$

$$m_j = m_l \pm \tfrac{1}{2}$$

$$\langle j\,m_j|\vec{S}\cdot\vec{L}|j\ m_j\rangle = \frac{\hbar^2}{2}\begin{cases} l & \text{اگر } j=l+\frac{1}{2} \\ -l-1 & \text{〃 } j=l-\frac{1}{2}\end{cases}$$

$\langle \vec{S}\cdot\vec{I}\rangle$

$$\{H_0, \vec{L}^2, L_3, \vec{S}^2, S_3, \vec{I}^2, I_3\} \quad |n\,l\,m_l\rangle|s,m_s\rangle|I,m_I\rangle \;\longrightarrow\; \{H_0, \vec{L}^2, L_3, \vec{F}^2, F_3, \vec{I}^2, \vec{S}^2\}$$

$$\vec{F} = \vec{S}+\vec{I}$$

$$I^2|I,m_I\rangle = \hbar^2 I(1+I)|I,m_I\rangle \qquad \vec{F}^2|F,m_F\rangle = \hbar^2F(F+1)|F,m_F\rangle$$

$$I_3|I,m_I\rangle = \hbar m_I|I,m_I\rangle \qquad F_3|F,m_F\rangle = \hbar m_F|F,m_F\rangle$$

$$\langle F,m_F|\vec{S}\cdot\vec{I}|F,m_F\rangle = \frac{\hbar^2}{2}\begin{cases} I & F=I+\frac{1}{2} \text{ اگر} \\ -I-1 & F=I-\frac{1}{2} \text{ 〃}\end{cases}$$

عملگر	اعداد کوانتومی
$\vec{L}\to\vec{I}$	$l\to I$
$\vec{S}\to\vec{S}$	$\frac{1}{2}=s \to s=\frac{1}{2}$
$\vec{J}^2\to\vec{F}^2$	$j\to F$
$J_3\to F_3$	$m_j\to m_F$

$$E_n^{(1)} = \langle H_4\rangle_{l=0} = -\frac{2}{3}\langle\vec{S}\cdot\vec{I}\rangle\gamma_e\gamma_N|R_{n0}(0)|^2$$

$$= -\frac{2}{3}\left(\frac{-eg_s}{2m_ec}\right)\left(\frac{Zeg_N}{2m_pc}\right)\left(\frac{4Z\alpha m_ec}{\hbar n}\right)^3\frac{\hbar^2}{2}\begin{cases} I & \text{اگر } F=I+\frac{1}{2} \\ -I-1 & \text{〃 } F=I-\frac{1}{2}\end{cases}$$

$$\Delta E_n^{(1)} = \frac{2}{3}g_sg_N\frac{m_e}{m_p}\frac{(Z\alpha)^4}{n^3}\frac{1}{2}m_ec^2\left(I-(-I-1)\right)$$

$$1\,Ry = \tfrac{1}{2}m_ec^2(Z\alpha)^2$$

$$= 5.7\times10^{-12}\,\frac{1}{n^3}\,\text{MeV} \xrightarrow{} J$$

$$J = Js\,\frac{1}{s}$$

$$E = h\nu$$

$n=1 \quad \rightarrow \quad 1420\ \text{MHz}$ ✓ $\quad \lambda = 21\ \text{cm}$

سیستم‌های N ذره‌ای

- فرمیون ها — اسپین نیم‌صحیح — الکترون - نوترون → $\frac{1}{2}, \frac{3}{2}, \ldots$

سیستم N ذره‌ای

- فرمیون ها — اسپین نیم‌صحیح — الکترون، پروتون: $\frac{1}{2}, \frac{3}{2}, \dots$
- بوزون‌ها — " عدد صحیح — $0, 1, 2, \dots$ — هیگز، فوتون، graviton

N ذره یکسان

سوال: تابع موج

$$P_{ij}\,\psi(1 \dots i \dots j \dots N) = \psi(1 \dots j \dots i, \dots N) \qquad 1 = (\vec{x}_1, \vec{\sigma}_1), \quad i = (\vec{x}_i, \vec{\sigma}_i)$$

$$\boxed{P_{ij}^2\,\psi(1 \dots i \dots j \dots N) = 1\;\psi(1, \dots i, \dots j \dots N)}$$

$$P_{ij}^2 = 1 \qquad P_{ij} = \pm 1$$

$$P_{ij}\,\psi_{S/A}(1 \dots i, \dots j, \dots N) = \psi(1 \dots j \dots i \dots N) = \pm\,\psi_{S/A}(1, \dots i, \dots j \dots N)$$

S متقارن

A پادمتقارن

$$\vec{S}_1 + \vec{S}_2 = \vec{S}$$

$$|s, m_s\rangle$$

$$|1,1\rangle$$

$$\rightarrow |1,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|\lambda_1\rangle_{(1)}|\lambda_2\rangle_{(2)} + |\lambda_2\rangle_{(1)}|\lambda_1\rangle_{(2)}\right) \quad \text{متقارن}$$

$$|1,-1\rangle$$

$$\rightarrow |0,0\rangle = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow - \downarrow\uparrow\right) = \frac{1}{\sqrt{2}}\left(|\lambda_1\rangle_{(1)}|\lambda_2\rangle_{(2)} - |\lambda_2\rangle_{(1)}|\lambda_1\rangle_{(2)}\right) \quad \text{پادمتقارن}$$

$$P_{12}\,|1,0\rangle = |1,0\rangle \qquad P_{12}\,|0,0\rangle = -|0,0\rangle$$

$|\lambda\rangle$ حالت تک ذره

$$|\lambda_1\rangle_{(1)} \otimes |\lambda_2\rangle_{(2)} \dots \otimes |\lambda_N\rangle_{(N)} \equiv |\lambda_1 \dots \lambda_N\rangle$$

حالت — ذره

بوزون‌ها: تابع موج کل کاملاً متقارن است تحت جابجائی هر دو جفت ذره

فرمیون‌ها: " " " پادمتقارن " " " "

بوزون‌ها: متقارن سازی تابع N ذره‌ای برای بوزون‌ها

$$|\lambda_1 \dots \lambda_N\rangle_+ = \frac{1}{\sqrt{N!}} \sum_{P \in S_N} (+1)^P\, P\,|\lambda_1 \dots \lambda_N\rangle$$

$$\boxed{|\lambda_1 \dots \lambda_N\rangle_+ = \frac{1}{\sqrt{N!}} \sum_{P} |\lambda_{P(1)} \dots \lambda_{P(N)}\rangle}$$

جایگشت

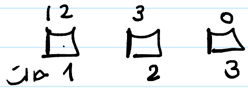
$$\begin{array}{c} \text{حالت} \\ \text{ذره} \end{array} \begin{pmatrix} 1 & 2 & 3 \\ 1 = P(1) & 2 = P(2) & 3 = P(3) \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 3 = P(1) & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

جایگشت زوج

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \end{pmatrix}$$

جایگشت فرد

$$\begin{array}{r} \text{ذره} \\ \text{حالت} \end{array} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}$$



عدد اشغال $n_1 = 2$ ←

$n_2 = 1$

$n_3 = 0$

زوج $\begin{pmatrix} 1 & 2 & 3 \\ \cancel{1} & \cancel{2} & \cancel{3} \\ 1 & 1 & 2 \end{pmatrix}_{*} \quad \begin{pmatrix} 1 & 2 & 3 \\ \cancel{3} & \cancel{1} & \cancel{2} \\ 2 & 1 & 1 \end{pmatrix}_{**} \quad \begin{pmatrix} 1 & 2 & 3 \\ \cancel{2} & \cancel{3} & \cancel{1} \\ 1 & 2 & 1 \end{pmatrix}_{***}$

فرد $\begin{pmatrix} 1 & 2 & 3 \\ \cancel{1} & \cancel{3} & \cancel{2} \\ 1 & 2 & 1 \end{pmatrix}_{***} \quad \begin{pmatrix} 1 & 2 & 3 \\ \cancel{3} & \cancel{2} & \cancel{1} \\ 2 & 1 & 1 \end{pmatrix}_{**} \quad \begin{pmatrix} 1 & 2 & 3 \\ \cancel{2} & 1 & \cancel{3} \\ 1 & 1 & 2 \end{pmatrix}_{*}$

$$|\lambda_1 \lambda_2 \lambda_3\rangle_+ = \frac{1}{\sqrt{3!}} \left(2\, |1\,1\,2\rangle + 2\, |2\,1\,1\rangle + 2\, |1\,2\,1\rangle \right)$$

$${}_+\langle \lambda_1 \lambda_2 \lambda_3 | \lambda_1 \lambda_2 \lambda_3 \rangle_+ = \frac{1}{3!} \left(2^2 + 2^2 + 2^2 \right) = 2$$

$\langle \acute{1}\, \acute{1}\, 2 | 1\,1\,2 \rangle = 1$

$\langle 1\,1\,2 | 2\,1\,1 \rangle = 0$

$\langle \uparrow \downarrow | \uparrow \downarrow \rangle = 1$

$\langle \uparrow \downarrow | \downarrow \uparrow \rangle = 0$

$$\frac{1}{\sqrt{2!}} |\lambda_1 \lambda_2 \lambda_3\rangle_+ = \frac{1}{\sqrt{\underset{2!}{n_1!}\, \underset{1}{n_2!}\, \underset{1}{n_3!}}} |\lambda_1 \lambda_2 \lambda_3\rangle_+$$

نرم به یک $\quad \widehat{|\lambda_1 \ldots \lambda_N\rangle_+} = \frac{1}{\sqrt{n_1!\, n_2! \ldots n_N!}} |\lambda_1 \ldots \lambda_N\rangle_+$