Lecture 23-QM_II (20/09/00)

Saturday, December 11, 2021 2:26 PM

$$H = H_0 + V(t)$$ اختلال وابسته به زمان

$$\begin{cases} t \geq t_0 & V(t) \neq 0 \\ t < t_0 & V(t) = 0 \end{cases}$$

تصویر برهمکنش (تصویر دیراک)

$$* \quad i\hbar \frac{\partial}{\partial t} \psi_I(\vec{x},t) = V_I(t)\, \psi_I(\vec{x},t)$$

$$V_I(t) = e^{iH_0 t/\hbar}\, V\, e^{-iH_0 t/\hbar}$$

$$\int_{t_0}^{t} dt'\, i\hbar \frac{\partial}{\partial t'} \psi_I(\vec{x},t') = \int_{t_0}^{t} dt'\, V_I(t')\, \psi_I(\vec{x},t')$$

$$\psi_I(\vec{x},t) = \psi_I(\vec{x},t_0) + \frac{1}{i\hbar}\int_{t_0}^{t} dt'\, V_I(t')\, \psi_I(\vec{x},t')$$

Neumann سری نویمن

$$\psi_I(\vec{x},t) = \psi_I(\vec{x},t_0) + \frac{1}{i\hbar}\int_{t_0}^{t} dt'\, V_I(t')\, \psi_I(\vec{x},t_0)$$
$$+ \left(\frac{1}{i\hbar}\right)^2 \int_{t_0}^{t} dt'\, V_I(t') \int_{t_0}^{t'} dt''\, V_I(t'')\, \psi_I(\vec{x},\cancel{t''}\ t_0) + \cdots$$

گذار اتمی در مرتبه اول اختلال

$$H_0 |n\rangle = E_n |n\rangle$$

$$|n\rangle \rightarrow e^{-iH_0 t/\hbar}|n\rangle = |n,t\rangle_s$$

حالت اولیه $\quad |m\rangle \rightarrow e^{-iH_0 t/\hbar}|m\rangle = |m,t\rangle_s$

$$|m,t\rangle_I = e^{+iH_0 t/\hbar}|m,t\rangle_s = |m\rangle$$

$$|\psi,t\rangle_s$$

$|m,t\rangle_I = |m\rangle$ $\qquad V(t) \qquad t \geq t_0$

$m \neq n$

$${}_s\langle n,t|\psi,t\rangle_s = \langle n|\underbrace{e^{+iH_0 t/\hbar}|\psi,t\rangle_s}_{|\psi,t\rangle_I} = \langle n|\psi,t\rangle_I$$

$$|m,t\rangle_I = |\psi,t_0\rangle_I = |m\rangle$$ (استفاده از سری نویمن در مرتبه اول)

$$\langle n|\psi,t\rangle_I = \underbrace{\cancel{\langle n|m\rangle}}_{\delta_{nm},\ n\neq m} + \frac{1}{i\hbar}\int_{t_0}^{t} dt'\, \langle n|V_I(t')|m\rangle$$

$H_0|n\rangle = E_n|n\rangle \qquad H_0|m\rangle = E_m|m\rangle$

$$= \frac{1}{i\hbar}\int_{t_0}^{t} dt'\, \langle n|e^{iH_0 t'/\hbar}\, V(t')\, e^{-iH_0 t/\hbar}|m\rangle$$

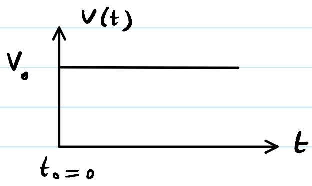
$$\boxed{\langle n|\psi,t\rangle_I = \frac{1}{i\hbar}\int_{t_0}^{t} dt'\, e^{i(E_n - E_m)t'/\hbar}\, \langle n|V(t')|m\rangle}$$

دامنه احتمال گذار

$$P_{m\to n}(t) = |\langle n|\psi,t\rangle_I|^2$$

$\uparrow V(t)$ $\qquad t>0 \qquad E_n - E_m$

$$P_{m\to n}(t) = \left| \langle n | \psi, t \rangle_I \right|^2$$



$$V(t) = V_0\,\theta(t) \qquad t \geq 0 \qquad \frac{E_n - E_m}{\hbar} = \omega_{nm}$$

$$\langle n | \psi, t \rangle_I = \frac{1}{i\hbar} \int_0^t dt'\, e^{i\omega_{nm} t'} \langle n | V_0 | m \rangle$$

$$= \frac{1}{i\hbar} \langle n | V_0 | m \rangle \frac{1}{i\omega_{nm}} \left(e^{i\omega_{nm} t} - 1 \right)$$

$$= \frac{1}{i\hbar} \langle n | V_0 | m \rangle \frac{e^{i\omega_{nm} t/2}}{\cancel{i}\omega_{nm}} \underbrace{\left(e^{i\omega_{nm} t/2} - e^{-i\omega_{nm} t/2} \right)}_{2\cancel{i} \sin \frac{\omega_{nm} t}{2}}$$

$$\langle n | \psi, t \rangle_I = \frac{1}{i\hbar} \langle n | V_0 | m \rangle\, e^{i\omega_{nm} t/2} \frac{\sin \frac{\omega_{nm} t}{2}}{\frac{\omega_{nm}}{2}}$$

$$P_{m\to n}(t) = \frac{1}{\hbar^2} \left| \langle n | V_0 | m \rangle \right|^2 \left(\frac{\sin \frac{\omega_{nm} t}{2}}{\frac{\omega_{nm}}{2}} \right)^2 \qquad \frac{\pi t}{\pi t}$$

✓ $$\delta_t(\alpha) = \frac{\sin^2 \alpha t}{\pi t \, \alpha^2} \xrightarrow{t\to\infty} \delta(\alpha)$$

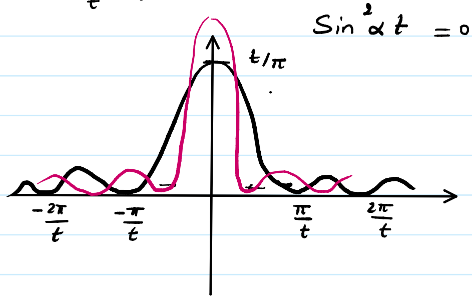
$$P_{m\to n}(t) = \frac{\pi t}{\hbar^2} \left| \langle n | V_0 | m \rangle \right|^2 \delta_t\left(\frac{\omega_{nm}}{2} \right)$$

$$\boxed{\text{آهنگ گذار} \quad \Gamma_{m\to n} = \lim_{t\to\infty} \frac{P_{m\to n}(t)}{t} = \frac{2\pi}{\hbar} \left| \langle n | V_0 | m \rangle \right|^2 \delta(E_n - E_m)}$$

اصل طلائی فرمی

$$\delta_t(\alpha) \Big|_{\alpha=0} = \frac{t^{\cancel{2}}}{\pi \cancel{t}} \frac{\sin^2 \alpha t}{\alpha^2 t^2} = \frac{t}{\pi} \xrightarrow[t\to\infty]{} \infty$$

$$\delta_t(\alpha) = 0 \qquad \sin^2 \alpha t = 0 \qquad \alpha t = n\pi \qquad \alpha = \frac{n\pi}{t} \qquad n = \pm 1, \pm 2$$



$$\lim_{t\to\infty} \int_{-\infty}^{+\infty} \delta_t(\alpha)\, d\alpha = 1$$

$$\lim_{t\to\infty} \int_{-\infty}^{+\infty} \underbrace{\delta_t(\alpha)}_{\delta(\alpha)} F(\alpha)\, d\alpha = F(0)$$

مثال ۲

$$V(t) = \theta(t) \left(F e^{-i\omega t} + F^{+} e^{+i\omega t} \right)$$

$$\langle n, t | \psi, t \rangle \underset{m \neq n}{=} \frac{1}{i\hbar} \int_0^t dt'\, e^{i\omega_{nm} t'} \langle n | V(t') | m \rangle$$

$$= \frac{1}{i\hbar} \int_0^t dt'\, e^{i(\omega_{nm} - \omega) t'} \langle n | F | m \rangle$$

$$+ \frac{1}{i\hbar} \int_0^t dt'\, e^{i(\omega_{nm} + \omega) t'} \langle n | F^{+} | m \rangle$$

$$\left| \langle n, t | \psi, t \rangle \right|^2 = \frac{\pi t}{\hbar^2} \left\{ \frac{\sin^2 \left(\frac{(\omega_{nm} - \omega) t}{2} \right)}{\pi t \left(\frac{\omega_{nm} - \omega}{2} \right)} \left| \langle n | F | m \rangle \right|^2 + \frac{\sin^2 \left(\frac{(\omega_{nm} + \omega) t}{2} \right)}{\pi t \left(\frac{\omega_{nm} + \omega}{2} \right)} \left| \langle n | F^{+} | m \rangle \right|^2 \right\} = P_{m\to n}(t)$$

$$\lim_{t\to\infty} \frac{P_{m\to n}(t)}{t} = \underbrace{\Gamma_{m\to n}}_{\text{آهنگ گذار}} = \frac{2\pi}{\hbar} \left\{ \delta(E_n - E_m - \hbar\omega) \left| \langle n | F | m \rangle \right|^2 + \delta(E_n - E_m + \hbar\omega) \left| \langle n | F^{+} | m \rangle \right|^2 \right\}$$

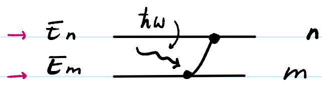
$$\lim_{t\to\infty} \frac{P_{m\to n}(t)}{t} = \underset{\text{آهنگ گذار}}{\Gamma_{m\to n}} = \frac{2\pi}{\hbar}\Big\{ \delta(E_n - E_m - \hbar\omega)\, |\langle n|F|m\rangle|^2 + \delta(E_n - E_m + \hbar\omega)\, |\langle n|F^{+}|m\rangle|^2 \Big\}$$

$$V(t) = \theta(t)\left\{ F e^{i\omega t} + F^{+} e^{-i\omega t} \right\}$$

جمله اول

$$\delta(E_n - E_m - \hbar\omega)$$

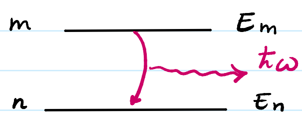
$$E_n - E_m = \hbar\omega > 0$$



جمله دوم

$$\delta(-E_n + E_m - \hbar\omega)$$

$$E_m - E_n = \hbar\omega > 0$$



In[48]:= $f[t\_, \alpha\_] := \frac{1}{\pi t}\left(\frac{\mathrm{Sin}[\alpha t]}{\alpha}\right)^2$

In[49]:= f[10, 10]

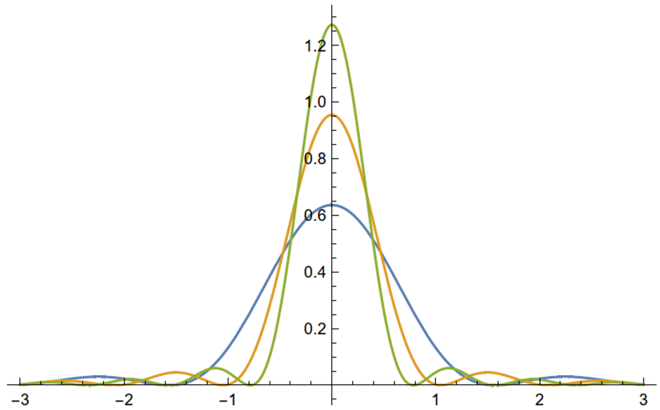
Out[49]= $\frac{\mathrm{Sin}[100]^2}{1000\,\pi}$

In[51]:= N[3 * π / 10]

Out[51]= 0.942478

In[59]:= Plot[{f[2, α], f[3, α], f[4, α]}, {α, -3, 3}, PlotRange → All]

Out[59]=



In[94]:= NIntegrate[f[1000, α], {α, -Infinity, Infinity}]

Out[94]= 1.

In[80]:= NIntegrate[f[1, α] * Cos[α], {α, -Infinity, Infinity}]

Out[80]= 0.5

In[81]:= NIntegrate[f[10, α] * Cos[α], {α, -Infinity, Infinity}]

Out[81]= 0.95

In[92]:= NIntegrate[f[$2 * 10^4$, α] * Cos[α], {α, -Infinity, Infinity}]

Out[92]= 0.999975