Lecture 25-QM_II (27/09/00)

Saturday, December 18, 2021 2:56 PM

$$H = H_0 + H_{int} + H_{rad}$$

$$H_0 = \sum_i \left(\frac{\vec{p}_i^{\,2}}{2m} + V(\vec{x}_i)\right)$$

$$H_{int} = V(t) = \int d^3x \left(-\frac{e}{c}\vec{J}(\vec{x})\cdot\vec{A}(x,t) + \frac{e^2}{2mc^2}\rho(\vec{x})\vec{A}^2(\vec{x},t) + \cancel{\rho(\vec{x})\varphi(\vec{x},t)}\right)$$

ρ_0 دریم ندارن

$$\rho(\vec{x}) = \sum_i \delta(\vec{x}-\vec{x}_i)$$

$$\vec{J}(\vec{x}) = \sum_i \left\{\frac{\vec{p}_i}{2m}, \delta(\vec{x}-\vec{x}_i)\right\}$$

$$\underset{\vec{J}(\vec{x},t)}{\rho(\vec{x},t)} = e^{iH_0t/\hbar}\underset{\vec{J}(\vec{x})}{\rho(\vec{x})}\, e^{-iH_0t/\hbar}$$

$$H_{rad} = \frac{1}{8\pi}\int d^3x\,(\vec{E}^2+\vec{B}^2) \overset{\cdots}{=} \sum_{\vec{k},\lambda}\hbar\omega_{\vec{k}}\,(\underbrace{a^+_{k,\lambda}a_{\vec{k},\lambda}} + \tfrac{1}{2})$$

رابطه پاشندگی میدان الکترومغناطیس

$$\boxed{\vec{A}(\vec{x},t) = \sum_{\vec{k},\lambda}\sqrt{\frac{2\pi\hbar c}{kV}}\left(a_{\vec{k},\lambda}\,\vec{\epsilon}_{\vec{k},\lambda}\,e^{i\vec{k}\cdot\vec{x}-i\omega_{\vec{k}}t} + a^+_{\vec{k},\lambda}\,\vec{\epsilon}^{\,*}_{\vec{k},\lambda}\,e^{-i\vec{k}\cdot\vec{x}+i\omega_{\vec{k}}t}\right)}$$

$$|i\rangle \to |f\rangle \qquad |m\rangle \to |n\rangle$$

گسیل خودبخود فوتون

$$\boxed{V(t) = \theta(t)\left(F e^{-i\omega t} + F^+ e^{+i\omega t}\right)} \;*$$

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar}\left\{\delta(E_n-E_m-\hbar\omega)\,|\langle n|F|m\rangle|^2 + \delta(E_n-E_m+\hbar\omega)\,|\langle n|F^+|m\rangle|^2\right\}$$

$$V(t) = \int d^3x\left(\underbrace{-\frac{e}{c}\vec{J}(\vec{x})\cdot\vec{A}(\vec{x},t)}_{1} + \cdots \vec{A}^2(\vec{x},t)\right)$$

$$a^+_{k,\lambda}\,|0\rangle_{\text{فوتون}} = |1_{\vec{k},\lambda}\rangle$$

$$a_{\vec{k},\lambda}\,|1_{\vec{k},\lambda}\rangle = |0\rangle_{\text{فوتون}}$$

$|i\rangle = |$ فوتون $\rangle \otimes |$ الکترون $\rangle$

$$|i\rangle = \underbrace{|0\rangle_{\text{فوتون}}}_{\text{Fock}} \otimes \underbrace{|m\rangle_{\text{الکترون}}}_{\text{Hilbert}} \qquad E_m$$

$$|f\rangle = |1_{\vec{k},\lambda}\rangle \otimes |n\rangle \qquad E_n$$

$$V_1(t) = \int d^3x\, \vec{J}(x)\cdot\vec{A}(\vec{x},t) =$$

$$= \sum_{k,\lambda}\sqrt{\frac{2\pi\hbar c}{kV}}\left(\frac{-e}{c}\right)\left\{\int d^3x\,\vec{J}(\vec{x})\cdot\vec{\epsilon}_{\vec{k},\lambda}\,a_{\vec{k},\lambda}\,e^{i\vec{k}\cdot\vec{x}}\,e^{-i\omega_{\vec{k}}t} + \int d^3x\,\vec{J}(\vec{x})\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}\,a^+_{\vec{k},\lambda}\,e^{-i\vec{k}\cdot\vec{x}+i\omega_k t}\right\}$$

$$V_1(t) = \sum_{k,\lambda}\left(F_{k,\lambda}e^{-i\omega_k t} + F^+_{k,\lambda}e^{+i\omega_k t}\right)$$

$$F_{\vec{k},\lambda} = \sqrt{\frac{2\pi\hbar c}{kV}}\left(\frac{-e}{c}\right)\vec{J}^{\,*}_{\vec{k}}\cdot\vec{\epsilon}_{\vec{k},\lambda}\,a_{\vec{k},\lambda}$$

$$F^+_{\vec{k},\lambda} = \sqrt{\frac{2\pi\hbar c}{kV}}\left(\frac{-e}{c}\right)\vec{J}_k\cdot\vec{\epsilon}^{\,*}_{k,\lambda}\,a^+_{\vec{k},\lambda}$$

$$F^{+}_{\vec k,\lambda} = \sqrt{\frac{2\pi\hbar c}{kV}}\left(\frac{-e}{c}\right) \vec J_k \cdot \vec\epsilon^{\,*}_{k,\lambda}\, a^{+}_{\vec k,\lambda}$$

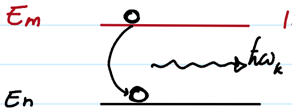
$$\underset{|m\rangle\otimes|0\rangle \to |n\rangle\otimes|1\rangle}{\Gamma_{i\to f;k,\lambda}} = \frac{2\pi}{\hbar}\Big\{\delta(E_n - E_m - \hbar\omega_{\vec k})\,\cancel{|\langle f|F_{\vec k,\lambda}|i\rangle|^2} + \delta(E_n - E_m + \hbar\omega_k)\,|\langle f|F^{+}_{\vec k,\lambda}|i\rangle|^2\Big\}$$

$\langle f|F|i\rangle \to$ فوتون$\langle 1|a_{k,\lambda}|0\rangle$فوتون $=0$

$\langle f|F^{+}|i\rangle$ فوتون$\langle 1|a^{+}_{k,\lambda}|0\rangle$فوتون $= \langle 1|1\rangle = 1$

الکترون$\langle n|$ فوتون$\langle 1|F^{+}_{\vec k,\lambda}|0\rangle$فوتون $|m\rangle$الکترون $= \dots \langle n|\vec J_k\cdot\vec\epsilon^{\,*}_{k,\lambda}|m\rangle$

$$\boxed{\Gamma_{i\to f;\vec k,\lambda} = \frac{2\pi}{\hbar}\left(\left(\frac{-e}{c}\right)\sqrt{\frac{2\pi\hbar c}{kV}}\right)^2 \delta(E_n - E_m + \hbar\omega_k)\,|\langle n|\vec J_k\cdot\vec\epsilon^{\,*}_{\vec k,\lambda}|m\rangle|^2}$$



$|m\rangle|0\rangle = |i\rangle$ $\qquad E_m = E_n + \hbar\omega_k$

$E_m - E_n = \hbar\omega_k$

$|n\rangle|1\rangle = |f\rangle$

توان تابش در زاویه فضایی $d\Omega$

$d\Omega = d(\cos\theta)\,d\varphi$

$\int d\Omega = 4\pi$

$$dP_\lambda = \sum_{\vec k\in d\Omega} \hbar\omega_k\, \Gamma_{i\to f;\vec k,\lambda}$$

زاویه فضایی

احتمال گذار / زمان

انرژی / زمان

توان

$$\sum_{\vec k\in d\Omega} \to d\Omega\int k^2dk\,\frac{V}{(2\pi)^3}$$

$d^3k\,\frac{V}{(2\pi)^3}$

$k^2dk\,d\Omega$

$$dP_\lambda = d\Omega\int k^2dk\,\frac{\cancel{V}}{(2\pi)^{\cancel{3}}}\,\hbar\omega_{\vec k}\,\frac{(\cancel{2\pi})^2 e^2}{\cancel{kV}\cancel{c}}\,\delta(E_n - E_m + \hbar\omega_k)\,|\langle n|\vec J_{\vec k}\cdot\vec\epsilon^{\,*}_{k,\lambda}|m\rangle|^2$$

$\cancel{\hbar k c}$ $\quad kc$ $\quad \underset{=0}{k}$

$\omega_{\vec k} = kc$

$= \delta\left(\left(\frac{E_n - E_m}{\hbar c} + k\right)\hbar c\right)$

$= \frac{1}{\cancel{\hbar c}}\,\delta\left(\frac{\omega_{nm}}{c} + k\right)$ $\qquad \omega_{nm} = \frac{E_n - E_m}{\hbar}$

$$= d\Omega\left(\frac{-\omega_{nm}}{c}\right)^2\frac{e^2}{2\pi c}\,|\langle n|\vec J_{\vec k=0}\cdot\vec\epsilon^{\,*}_{\vec k,\lambda}|m\rangle|^2$$

$$\boxed{\frac{dP_\lambda}{d\Omega} = \frac{e^2}{2\pi c^3}\,\omega_{nm}^2\,|\langle n|\vec J_{\underset{0}{\vec k}}\cdot\vec\epsilon^{\,*}_{\vec k,\lambda}|m\rangle|^2}$$

$$\vec J_k = \int \vec J(\vec x)\cdot \underbrace{e^{-i\vec k\cdot\vec x}}\,d^3x = \int \vec J(\vec x)\,d^3x - i\int \vec J(\vec x)(\vec k\cdot\vec x)\,d^3x + \dots$$

بسط چند قطبی

$(1 - i\vec k\cdot\vec x + \frac{1}{2}(-i\vec k\cdot\vec x)^2 + \dots)$

$\vec J_0$

تقریب دوقطبی الکتریکی

zero mode

تقریب دوقطبی مغناطیسی