Lecture 26-QM_II (29/09/00)

Monday, December 20, 2021 3:01 PM

$$\vec{A}(\vec{x},t)=\sum_{k,\lambda}\sqrt{\frac{2\pi\hbar c}{kV}}\left(a_{k,\lambda}\,\vec{\epsilon}_{k,\lambda}\,e^{i\vec{k}\cdot\vec{x}-i\omega_k t}+a^{+}_{k,\lambda}\,\vec{\epsilon}^{\,*}_{\vec{k},\lambda}\,e^{-i\vec{k}\cdot\vec{x}+i\omega_k t}\right)$$

$$V(t)=\frac{e}{c}\int d^3x\;\vec{J}(\vec{x})\cdot\vec{A}(\vec{x},t)+\dots$$

$$=\sum_{\vec{k},\lambda}\left(F_{k,\lambda}\,e^{-i\omega_k t}+F^{+}_{\vec{k},\lambda}\,e^{+i\omega_k t}\right)$$

$$\Gamma_{i\to f}=\frac{(2\pi)^2e^2}{kVc}\,\delta(E_n-E_m+\hbar\omega_k)\,\left|\langle n|\vec{J}_{\vec{k}}\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}|m\rangle\right|^2$$

E_m, γ, E_n

$E_m-E_n\neq 0 \quad >0$

$$|0\rangle|m\rangle\;\to\;|1\rangle|n\rangle$$

$$\langle 1|\,a^{+}_{k,\lambda}|0\rangle=1$$

$$\omega_k=kc \qquad k=|\vec{k}|$$

توان تابش $\quad dP_\lambda=\sum_{\vec{k}\in d\Omega_k}\hbar\omega_k\,\Gamma_{i\to f}=$

$$\sum_{\vec{k}\in d\Omega}=d^3k\,\frac{V}{(2\pi)^3}=d\Omega\;k^2dk\,\frac{V}{(2\pi)^3}$$

$$\boxed{\frac{dP_\lambda}{d\Omega}=\frac{e^2\omega_{nm}^2}{2\pi c^3}\left|\langle n|\vec{J}_0\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}|m\rangle\right|^2}$$

$$\vec{J}_k=\int d^3x\;\vec{J}(\vec{x})\,e^{-i\vec{k}\cdot\vec{x}}=\int d^3x\;\vec{J}(\vec{x})+\dots=\vec{J}_0+\dots$$

multipole exp. $\qquad$ $\underline{\underline{E1}}$ $\qquad$ zero mode

$$\vec{J}(\vec{x})=\frac{1}{2m}\sum_i\{\vec{p}_i,\;\delta(\vec{x}-\vec{x}_i)\}$$

$$\vec{J}_0=\int d^3x\,\frac{1}{2m}\sum_i\left(\vec{p}_i\,\delta(\vec{x}-\vec{x}_i)+\delta(\vec{x}-\vec{x}_i)\vec{p}_i\right)=\frac{1}{m}\sum_i\vec{p}_i=\frac{\vec{P}}{m}\quad *$$

$$\vec{R}_{cm}=\frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}=\frac{m\sum_i\vec{r}_i}{N\,m}=\frac{1}{N}\left(\sum_i\vec{r}_i\right)=\frac{1}{N}\underline{\underline{\vec{X}}}$$

$$\frac{d\vec{X}}{dt}=\frac{i}{\hbar}[H_0,\vec{X}]$$

$$H_0=\sum_i\frac{p_i^2}{2m}+V(\vec{x}) \qquad V(\vec{x})=-\frac{Ze^2}{r_i}$$

$$\frac{d\vec{X}}{dt}=\sum_i\frac{d\vec{r}_i}{dt}=\sum_i\frac{1}{m}\vec{p}_i\underset{*}{=}\vec{J}_0=\frac{i}{\hbar}[H_0,\vec{X}]$$

$$\frac{dP_\lambda}{d\Omega}=\frac{e^2\omega_{nm}^2}{2\pi c^3}\left|\langle n|\vec{J}_0\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}|m\rangle\right|^2$$

$$=\frac{e^2\omega_{nm}^2}{2\pi c^3\hbar^2}\left|\langle n|\,[H_0,\vec{X}]\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}|m\rangle\right|^2$$

$$\left|\langle n|\,H_0\vec{X}\cdot\vec{\epsilon}^{\,*}-\vec{X}\cdot\vec{\epsilon}_{\vec{k},\lambda}H_0|m\rangle\right|^2$$

$$H_0|n\rangle=E_n|n\rangle \qquad H_0|m\rangle=E_m|m\rangle$$

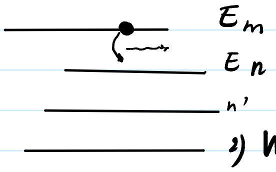
$$=\frac{e^2\omega_{nm}^2}{2\pi c^3\hbar^2}\underset{=\hbar^2\omega_{nm}^2}{(E_n-E_m)^2}\left|\langle n|\vec{X}\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}|m\rangle\right|^2$$

$$=\frac{e^2\omega_{nm}^4}{2\pi c^3}\underline{\left|\langle n|\vec{X}\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}|m\rangle\right|^2}$$ توان تابش دوقطبی الکتریکی

دوقطبی الکتریکی $\quad e\,\vec{d}_{nm}=e\langle n|\vec{x}|m\rangle \qquad \left|\vec{d}_{nm}\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda}\right|^2$

$$= \frac{\qquad}{2\pi c^3} \quad \underbrace{|\langle n | x \cdot \epsilon_{\vec{k},\lambda} | m \rangle|}$$ ول با تقریب دوقطبی الکتریک

دوقطبی الکتریک $\quad e\,\vec{d}_{nm} \equiv e\langle n | \vec{x} | m \rangle \qquad |\vec{d}_{nm} \cdot \vec{\epsilon}^{\,*}_{\vec{k},\lambda}|^2$



1) $$\underset{\smile}{d\omega_\lambda} = \sum_{\vec{k}\in d\Omega} T_{m\to n,\vec{k},\lambda} \checkmark \qquad \frac{\text{احتمال}}{\text{زمان}}$$

2) $$W_{nm} = \sum_{\lambda=1,2} \int d\omega_\lambda = \text{احتمال کل گسیل فوتون در واحد زمان}$$ از حالت $n \leftarrow m$

3) $$\tau = \frac{1}{\sum_n W_{nm}}$$ طول عمر حالت m ام.

1) $$\sum_{\vec{k}\in d\Omega} = d\Omega \int k^2 dk \frac{V}{(2\pi)^3}$$

$$d\omega_\lambda = d\Omega \int \cancel{k^2} dk \frac{V}{\cancel{(2\pi)^3}} \frac{\cancel{(2\pi)^2} e^2}{\cancel{k}cV} \delta\left(\underbrace{E_n - E_m + \hbar kc}\right) |\langle n | \vec{J}_0 \cdot \vec{\epsilon}^{\,*}_{\vec{k},\lambda} | m \rangle|^2$$

$$\frac{1}{\hbar c}\,\delta\left(\frac{\omega_{nm}}{c} + k\right)$$

$$= d\Omega \frac{e^2 \omega_{mn}}{2\pi\hbar c^3} |\langle n | \vec{J}_0 \cdot \vec{\epsilon}^{\,*}_{\vec{k},\lambda} | m \rangle|^2$$

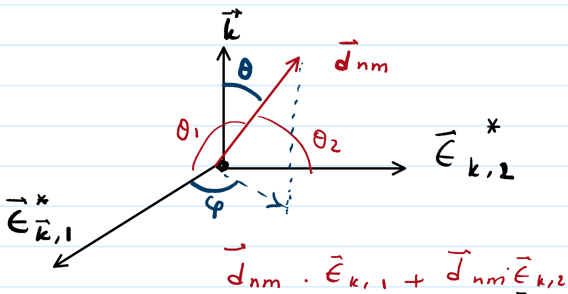
$$\vec{J}_0 \to \frac{i}{\hbar}[H_0, \vec{X}]$$

$$d\omega_\lambda = d\Omega \frac{e^2 \omega_{mn}^3}{2\pi\hbar c^3} |\underset{\smile}{\vec{d}_{nm} \cdot \vec{\epsilon}^{\,*}_{\vec{k},\lambda}}|^2$$

2) $$\sum_\lambda \int d\omega_\lambda$$

$\cos\theta_1 = \sin\theta\cos\varphi$ ✓

$\cos\theta_2 = \sin\theta\sin\varphi$ ✓



$$\vec{d}_{nm} \cdot \vec{\epsilon}_{k,1} + \vec{d}_{nm} \cdot \vec{\epsilon}_{k,2}$$

$$\sum_\lambda |\vec{d}_{nm} \cdot \vec{\epsilon}^{\,*}_{k,\lambda}|^2 = |\vec{d}_{nm}|^2 (\cos^2\theta_1 + \cos^2\theta_2)$$

$$\sum_\lambda |\vec{d}_{nm} \cdot \vec{\epsilon}^{\,*}_{\vec{k},\lambda}|^2 = |\vec{d}_{nm}|^2 \sin^2\theta\, (\underbrace{\cos^2\varphi + \sin^2\varphi}_{1})$$

$$= |\vec{d}_{nm}|^2 (1 - \cos^2\theta)$$

$$\sum_\lambda \int d\omega_\lambda = \sum_\lambda \int \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \int_{-1}^{1} \frac{e^2\omega_{mn}^3}{2\pi\hbar c^3} \underbrace{|\vec{d}_{nm} \cdot \vec{\epsilon}_{\vec{k},\lambda}|^2}_{|\vec{d}_{nm}|^2 (1-\cos^2\theta)} \quad d(\cos\theta)$$

$$\cos\theta - \frac{\cos^3\theta}{3}\Big|_{-1}^{1} = 2 - \frac{2}{3} = \frac{4}{3}$$

$$\boxed{\sum_\lambda \int d\omega_\lambda = \frac{4}{3} \frac{e^2\omega_{mn}^3}{\hbar c^3} |\vec{d}_{nm}|^2} = W_{mn}$$

$$\tau = \frac{1}{\sum_n W_{mn}} \checkmark$$ طول عمر تراز m ام

$$|\langle n | \underset{\smile}{\vec{J}_{\vec{k}}} \cdot \vec{\epsilon}^{\,*}_{\vec{k},\lambda} | m \rangle|^2$$

$$\int d^3x\, \underset{\smile}{e^{-i\vec{k}\vec{x}}}\, \vec{J}(x) = \vec{J}_k = \underset{\smile}{\vec{J}_0} + (-i)\int d^3x\, \underset{\smile}{(\vec{k}\cdot\vec{x})}\, \vec{J}(\vec{x}) \cdot \vec{\epsilon}^{\,*}_{\vec{k},\lambda} + \cdots$$

$$\int d^3x\, e^{-i\vec{k}\vec{x}}\, \vec{J}(x) = \vec{J}_k = \vec{J}_0 + (-i)\int d^3x \, (\vec{k}\cdot\vec{x})\, \vec{J}(\vec{x})\cdot\vec{\epsilon}^{\,*}_{\vec{k},\lambda} + \cdots$$