Lecture 3-QM_II (03/07/00)

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$$L = \frac{1}{2} m \dot{x}_i^2 - q\varphi + \frac{q}{c} \vec{A}\cdot\vec{v}$$

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = 0 \qquad m\ddot{\vec{x}} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right) = \vec{F}_L$$

$$H = \frac{1}{2m}\left(\vec{P} - \frac{q}{c}\vec{A}\right)^2 + q\varphi \qquad \vec{\nabla}\cdot\vec{A} = 0, \quad \varphi = 0$$

$$|\psi|^2$$

$$\frac{\partial H}{\partial x_i} = -\dot{P}_i$$

$$\frac{\partial H}{\partial P_i} = \dot{x}_i = \frac{1}{m}\left(P_i - \frac{q}{c}A_i\right)$$

تکانه جنبشی

$$\underbrace{m\dot{\vec{x}}} = \vec{P} - \frac{q}{c}\vec{A}$$

تکانه کانونیک

نگاشت کوانتومی

$$\vec{P} \rightarrow \frac{\hbar}{i}\vec{\nabla} \qquad \vec{x} \rightarrow \vec{x}$$

$$\vec{A}(\vec{x},t) \rightarrow \vec{A}(\vec{x},t)$$

$$\varphi(\vec{x},t) \rightarrow \varphi(\vec{x},t)$$

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = \left(\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} - \frac{q}{c}\vec{A}\right)^2 + q\varphi\right)\psi(\vec{x},t)}$$

$$\psi(\vec{x},t) = e^{-\frac{i}{\hbar}Et}\cdot\psi(\vec{x})$$

معادله ویژه مقداری انرژی

$$\left(\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} - \frac{q}{c}\vec{A}(\vec{x})\right)^2 + q\varphi(\vec{x})\right)\psi(\vec{x}) = E\psi(\vec{x})$$

$$-\vec{\nabla}\varphi - \cancel{\frac{1}{c}\frac{\partial}{\partial t}\vec{A}} = \vec{E} = 0$$

$$\vec{\nabla}\times\vec{A} = \vec{B}$$

$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$$

تکانه کانونیک

$$P_i = \frac{\partial L}{\partial \dot{x}_i}$$

تکانه جنبشی

$$m\dot{\vec{x}} = \vec{P} - \frac{q}{c}\vec{A}$$

$$[A_i, A_j] = 0$$

$$[P_i, P_j] = 0$$

$$[x_i, x_j] = 0$$

$$[m\dot{x}_i, m\dot{x}_j] \neq 0$$

$$[m\dot{x}_i, m\dot{x}_j] = \left[P_i - \frac{q}{c}A_i, P_j - \frac{q}{c}A_j\right]$$

$$= -\frac{q}{c}\underset{(a)}{[P_i, A_j]} - \frac{q}{c}\underset{(b)}{[A_i, P_j]}$$

a) $$[P_i, A_j] = \frac{\hbar}{i}[\partial_i, A_j] = \frac{\hbar}{i}\left(\partial_i A_j + \cancel{A_j\partial_i} - \cancel{A_j\partial_i}\right) = \frac{\hbar}{i}\partial_i A_j$$

$$[\partial_i, A_j]f = \partial_i(A_j f) - A_j\partial_i f$$
$$= (\partial_i A_j)f + \cancel{A_j\partial_i f} - \cancel{A_j\partial_i f}$$

b) $$[A_i, P_j] = -\frac{\hbar}{i}\partial_j A_i$$

$$\curvearrowright [m\dot{x}_i, m\dot{x}_j] = -\frac{q}{c}\frac{\hbar}{i}\underbrace{(\partial_i A_j - \partial_j A_i)}_{F_{ij} = \epsilon_{ijk}B_k} = +\frac{iq\hbar}{c}\epsilon_{ijk}B_k$$

$$[P_i, P_j] = 0$$

$$\left(\frac{1}{2m} \left(\frac{\hbar}{i}\vec{\nabla} - \frac{q}{c}\vec{A} \right)^2 + q\varphi \right) \psi(\vec{x}) = E\,\psi(\vec{x})$$

پیمانه کولن $\quad \varphi = 0 \quad \vec{\nabla}\cdot\vec{A} = 0$

$$\frac{1}{2m}\left(-\hbar^2 \vec{\nabla}^2\psi + \frac{q^2}{c^2}\vec{A}^2\psi - \frac{q}{c}\frac{\hbar}{i}\underbrace{(\vec{\nabla}\cdot\vec{A})}_{=0}\psi - 2\frac{q}{c}\frac{\hbar}{i}\vec{A}\cdot\vec{\nabla}\psi \right) = E\psi$$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi \underbrace{- \frac{q}{cm}\frac{\hbar}{i}\vec{A}\cdot\vec{\nabla}\psi}_{①} \underbrace{+ \frac{q^2}{2mc^2}\vec{A}^2\psi}_{②} = E\psi}$$

① میدان مغناطیسی همگن $\quad \vec{B} = B\hat{e}_z = B(0,0,1)$ در

$$\vec{A} = -\frac{1}{2}\vec{x}\times\vec{B} = -\frac{1}{2}B(y,-x,0)$$ پیمانه متقارن

$$\vec{\nabla}\times\vec{A} = \vec{B} = B\hat{e}_z$$

$$-\frac{q}{mc}\frac{\hbar}{i}\vec{A}\cdot\vec{\nabla}\psi = +\frac{q}{2mc}\underbrace{(\vec{x}\times\vec{B})}_{B}\cdot\vec{p}\,\psi \qquad \frac{\partial}{\partial x_k}$$

$$= \frac{q}{2mc}\epsilon_{ijk}\,x_i B_j p_k\,\psi$$

$$= -\frac{q}{2mc}\underbrace{\epsilon_{ikj}\,x_i p_k}_{L_j} B_j\,\psi$$

$$= -\frac{q\hbar}{2mc}\frac{\vec{L}}{\hbar}\cdot\vec{B}\,\psi$$

الکترون Bohr magneton $\quad \mu_B = \dfrac{e\hbar}{2m_e c} \Rightarrow ① = \mu_B \dfrac{\vec{L}}{\hbar}\cdot\vec{B}\,\psi$

پارامغناطیس $q = -e$

Magnetic moment $\quad \vec{\mu} = \mu_B \dfrac{\vec{L}}{\hbar}$

$$① = \vec{\mu}\cdot\vec{B}\,\psi = \mu_z B\,\psi \qquad \chi > 0$$

$$= \mu_B \frac{L_z}{\hbar}B\,\psi$$

دیامغناطیس ② $\quad \dfrac{q^2}{2mc^2}\vec{A}^2\psi = \dfrac{q^2}{8mc^2}B^2\underbrace{(x^2+y^2)}_{=\rho^2}$

$\vec{A} = -\frac{1}{2}B(y,-x,0)$

$x = \rho\cos\varphi$
$y = \rho\sin\varphi$
$z = z$

بدون بعد $\quad \dfrac{(2)}{(1)} = \dfrac{\dfrac{q^2}{8mc^2}B^2\langle\rho^2\rangle}{-\dfrac{q}{2mc}\langle\vec{B}\cdot\vec{L}\rangle} = \dfrac{1}{4}\alpha\dfrac{B}{\dfrac{e}{a_0^2}} = \underbrace{1.1\times10^{-10}(G^{-1})\,B}$

$\langle\rho^2\rangle \sim a_0^2 \qquad q = -e$

$L_z|nlm_l\rangle = \hbar m_l|nlm_l\rangle \qquad BL_z \to B\hbar m_l \sim B\hbar \qquad 1T = 10^4 G$

$a_0 = \dfrac{\hbar}{mc\alpha} \qquad \alpha = \dfrac{e^2}{\hbar c}$

$$\frac{(2)}{(1)} \sim 1.1\times10^{-10}\,B$$

$\dfrac{(2)}{(1)} = 1.1\times10^{-4}$ نسبتاً ضعیف $\quad 100T = 10^6 G$ •

$$(2) \ll (1)$$

$B \sim 10^{16} - 10^{18}$ G ستاره نوترونی •
$B \sim 0.6$ G میدان مغناطیسی زمین

• ستاره نوترونی $B \sim 10^{16} - 10^{18}\ G$

میدان مغناطیسی زمین $B \sim 0.6\ G$

$$\frac{(2)}{(1)} = 1.1 \times 10^{-10}\, G^{-1} \times 10^{16}\ G \sim 10^{6}$$

$$(2) \gg (1)$$

در جهت تقویت ۲

$$\Delta E = -\vec{\mu} \cdot \vec{B} = -\chi V B^2 > 0 \quad \begin{cases} \chi > 0 & \text{پارامغناطیس} \\ \chi < 0 & \text{دیامغناطیس} \end{cases}$$

$$\vec{\mu} = \chi V \vec{B}$$

پذیرفتاری مغناطیسی

$\underset{\sim}{\chi < 0}$

Normal Zeeman Effect. اثر نرمال زیمن

$$H = H_0 + H_{magn}$$

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{r}$$

$q = -e$

$$H_{magn} = \frac{e}{2mc} B L_z$$

فرض میدان مغناطیسی ضعیف

$$[H_0, L_z] = 0$$

$$H_0 |n \ell m_\ell\rangle = E_n |n \ell m_\ell\rangle$$

اتم هیدروژن ساده

$$E_n = -1Ry \frac{1}{n^2}$$

1 Rydberg $=$

$1\ Ry = \frac{1}{2} mc^2\alpha^2 \sim 13.6\,eV$

$$n = 1, 2, 3, \ldots$$

$$\ell = 0, 1, \ldots n-1$$

$$-\ell \leq m_\ell \leq \ell$$

$$H |n \ell m_\ell\rangle = E |n \ell m_\ell\rangle$$

$$\left(H_0 + \frac{e}{2mc} B L_z\right)|n \ell m_\ell\rangle = E |n \ell m_\ell\rangle$$

$$\left(E_n + \frac{eB\hbar}{2mc} m_\ell\right)|n \ell m_\ell\rangle = E_{n, m_\ell} |n \ell m_\ell\rangle$$

$$E_{n, m_\ell} = E_n + \underbrace{\frac{eB\hbar}{2mc}}_{\hbar\omega_L} \underset{\sim}{m_\ell}$$

$$\hbar\omega_L = \hbar \frac{eB}{2mc} \rightarrow$$ فرکانس لارمور Larmor frequency

$\ell = 2$: $m_\ell = 2$, $m_\ell = +1$, $m_\ell = 0$, $m_\ell = -1$, $m_\ell = -2$ $\qquad m_\ell = 0, \pm 1, \pm 2$

$\ell = 1$: $m_\ell = 1$, $m_\ell = 0$, $m_\ell = -1$ ($\hbar\omega_L$) $\qquad m_\ell = 0, \pm 1$

$\underset{\sim}{n = 3}$

$\ell = 0$ $\qquad m_\ell = 0$