

$$L = \frac{1}{2} m \dot{x}_i^2 - q\varphi + \frac{q}{c} \vec{A} \cdot \vec{v}$$

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} = 0 \quad m \ddot{\vec{x}} = q \left( \vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) = \vec{F}_L$$

$$H = \frac{1}{2m} \left( \vec{p} - \frac{q}{c} \vec{A} \right)^2 + q\varphi \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\frac{\partial H}{\partial x_i} = -\dot{p}_i$$

$$|\psi|^2$$

$$\frac{\partial H}{\partial p_i} = \dot{x}_i = \frac{1}{m} \left( p_i - \frac{q}{c} A_i \right)$$

$$m \ddot{\vec{x}} = \vec{p} - \frac{q}{c} \vec{A}$$

لانه ما زنیب  
لانه ما زنیب

$$\vec{p} \rightarrow \frac{\hbar}{i} \vec{\nabla}$$

$$\begin{aligned} \vec{x} &\rightarrow \vec{x} \\ \vec{A}(\vec{x}, t) &\rightarrow \vec{A}(\vec{x}, t) \\ \varphi(\vec{x}, t) &\rightarrow \varphi(\vec{x}, t) \end{aligned}$$

صائب رانتری

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t) = \left( \frac{1}{2m} \left( \frac{\hbar}{i} \vec{\nabla} - \frac{q}{c} \vec{A} \right)^2 + q\varphi \right) \psi(\vec{x}, t)$$

$$\psi(\vec{x}, t) = e^{-\frac{i}{\hbar} Et} \cdot \psi(\vec{x})$$

ساده رانتری بنداری رانتری

$$\left( \frac{1}{2m} \left( \frac{\hbar}{i} \vec{\nabla} - \frac{q}{c} \vec{A}(\vec{x}) \right)^2 + q\varphi(\vec{x}) \right) \psi(\vec{x}) = E \psi(\vec{x})$$

$$[\hat{x}_i, \hat{p}_j] = i\hbar \delta_{ij}$$

لانه ما زنیب

$$m \ddot{\vec{x}} = \vec{p} - \frac{q}{c} \vec{A}$$

$$[m\dot{x}_i, m\dot{x}_j] \neq 0$$

$$p_i = \frac{\partial L}{\partial \dot{x}_i}$$

$$\begin{aligned} -\vec{\nabla} \varphi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} = \vec{E} &\neq 0 \\ \vec{\nabla} \times \vec{A} = \vec{B} & \end{aligned}$$

$$[A_i, A_j] = 0$$

$$[p_i, p_j] = 0$$

$$[x_i, x_j] = 0$$

$$[m\dot{x}_i, m\dot{x}_j] = [p_i - \frac{q}{c} A_i, p_j - \frac{q}{c} A_j]$$

$$= -\frac{q}{c} [p_i, A_j] - \frac{q}{c} [A_i, p_j]$$

$$a) [p_i, A_j] = \frac{\hbar}{i} [\partial_i, A_j] = \frac{\hbar}{i} (\partial_i A_j + A_j \partial_i - A_j \partial_i) = \frac{\hbar}{i} \partial_i A_j$$

$$[\partial_i, A_j] \neq 0 = \partial_i (A_j f) - A_j \partial_i f = (\partial_i A_j) f + A_j \partial_i f - A_j \partial_i f$$

$$b) [A_i, p_j] = -\frac{\hbar}{i} \partial_j A_i$$

$$\rightarrow [m\dot{x}_i, m\dot{x}_j] = -\frac{q}{c} \frac{\hbar}{i} (\partial_i A_j - \partial_j A_i) = +\frac{iq\hbar}{c} \epsilon_{ijk} B_k$$

$$F_{ij} = \epsilon_{ijk} B_k$$

$$[p_i, p_j] = 0$$

$$\left( \frac{1}{2m} \left( \frac{\hbar}{i} \vec{\nabla} - \frac{q}{c} \vec{A} \right)^2 + q\psi \right) \psi(\vec{x}) = E \psi(\vec{x})$$

$\psi = 0$ 
 $\vec{\nabla} \cdot \vec{A} = 0$

$$\frac{1}{2m} \left( -\hbar^2 \nabla^2 \psi + \frac{q^2}{c^2} \vec{A}^2 \psi - \frac{q}{c} \frac{\hbar}{i} (\vec{\nabla} \cdot \vec{A}) \psi - 2 \frac{q}{c} \frac{\hbar}{i} \vec{A} \cdot \vec{\nabla} \psi \right) = E \psi$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi - \frac{q}{cm} \frac{\hbar}{i} \vec{A} \cdot \vec{\nabla} \psi + \frac{q^2}{2mc^2} \vec{A}^2 \psi = E \psi$$

①  $\vec{A} = -\frac{1}{2} \vec{x} \times \vec{B} = -\frac{1}{2} B (y, -x, 0)$

$$\vec{\nabla} \times \vec{A} = \vec{B} = B \hat{e}_z$$

$$-\frac{q}{mc} \frac{\hbar}{i} \vec{A} \cdot \vec{\nabla} \psi = + \frac{q}{2mc} (\vec{x} \times \vec{B}) \cdot \vec{p} \psi$$

$$= \frac{q}{2mc} \epsilon_{ijk} x_i B_j p_k \psi$$

$$= -\frac{q}{2mc} \epsilon_{ikj} x_i p_k B_j \psi$$

$$= -\frac{q\hbar}{2mc} \frac{\vec{L}}{\hbar} \cdot \vec{B} \psi$$

Bohr magneton  $\mu_B = \frac{e\hbar}{2m_e c}$   
 Magnetic moment  $\vec{\mu} = \mu_B \frac{\vec{L}}{\hbar}$

① =  $\mu_B \frac{\vec{L}}{\hbar} \cdot \vec{B} \psi$   
 ① =  $\frac{\vec{\mu} \cdot \vec{B}}{\mu_B} \psi$   
 =  $\mu_B \frac{L_z}{\hbar} B \psi$

②  $\frac{q^2}{2mc^2} \vec{A}^2 \psi = \frac{q^2}{8mc^2} B^2 (x^2 + y^2) \psi$

$\vec{A} = -\frac{1}{2} B (y, -x, 0)$

$x = \rho \cos \varphi$   
 $y = \rho \sin \varphi$   
 $z = z$

$$\frac{(2)}{(1)} = \frac{\frac{q^2}{8mc^2} B^2 \langle \rho^2 \rangle}{-\frac{q}{2mc} \langle \vec{B} \cdot \vec{L} \rangle} = \frac{1}{4} \alpha \frac{B}{\frac{e}{a_0^2}} = 1.1 \times 10^{-10} (G^{-1}) B$$

$L_z |nlm\rangle = \hbar m_l |nlm\rangle$   
 $B L_z \rightarrow B \hbar m_l \sim B \hbar$   
 $a_0 = \frac{\hbar}{mc\alpha}$   
 $\alpha = \frac{e^2}{\hbar c}$   
 $1T = 10^4 G$

$\frac{(2)}{(1)} \sim 1.1 \times 10^{-10} B$

$\frac{(2)}{(1)} = 1.1 \times 10^{-4}$        $100T = 10^6 G$

(2)  $\ll$  (1)

$B \sim 10^{16} - 10^{18} G$   
 $B \sim 0.6 G$

ستاره نوامبر  
میدان مغناطیسی  
 $B \sim 10^{16} - 10^{18} \text{ G}$   
 $B \sim 0.6 \text{ G}$

$$\frac{(2)}{(1)} = \frac{1.1 \times 10^{-10} \text{ G}^{-1} \times 10^{16} \text{ G}}{\sim 10^6}$$

(2)  $\gg$  (1)

$$\Delta E = -\vec{\mu} \cdot \vec{B} = -\chi \sqrt{B^2} \begin{cases} \chi > 0 & \text{پارامگنیتس} \\ \chi < 0 & \text{دیامگنیتس} \end{cases}$$

$\vec{\mu} = \chi \sqrt{B^2}$   
پارامگنیتس  
دیامگنیتس

Normal Zeeman Effect.

اثر نرمال زیمان

$$H = H_0 + H_{\text{magn}}$$

$$H_0 = -\frac{\hbar^2}{2m} \nabla^2 - \frac{Ze^2}{r}$$

$q = -e$

$$H_{\text{magn}} = \frac{e}{2mc} B L_z$$

فرض میدان مغناطیسی ضعیف

$$[H_0, L_z] = 0$$

$$H_0 |n l m_l\rangle = E_n |n l m_l\rangle$$

اتم هیدروژن ساده

$$E_n = -1 \text{ Ry} \frac{1}{n^2}$$

1 Rydberg =  $\frac{1}{2} m c^2 \alpha^2 \sim 13.6 \text{ eV}$

$n = 1, 2, 3, \dots$   
 $l = 0, 1, \dots, n-1$

$$-l \leq m_l \leq l$$

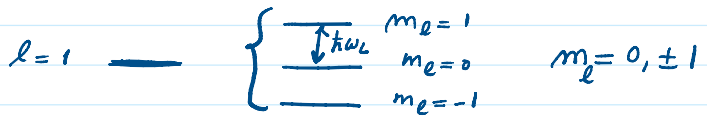
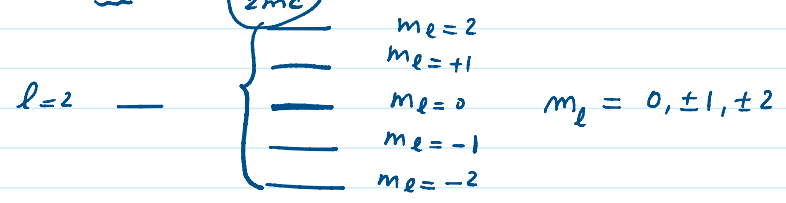
$$H |n l m_l\rangle = E |n l m_l\rangle$$

$$\left( H_0 + \frac{e}{2mc} B L_z \right) |n l m_l\rangle = E |n l m_l\rangle$$

$$\left( E_n + \frac{eB\hbar}{2mc} m_l \right) |n l m_l\rangle = E_{n, m_l} |n l m_l\rangle$$

$$E_{n, m_l} = E_n + \underbrace{\frac{eB\hbar}{2mc}}_{\hbar \omega_L} m_l$$

$\hbar \omega_L = \hbar \frac{eB}{2mc}$  → فرکانس لارمور Larmor frequency



$l=0$  — —  $m_l = 0$

