Lecture 4-QM_II (10/07/00)

Saturday, October 2, 2021 2:42 PM

$$\left(\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+q\varphi+V(\vec{x})\right)\psi(\vec{x})=E\psi(\vec{x})$$

$$H_0=\frac{\vec{p}^2}{2m_e}-\frac{Ze^2}{r}$$

$\vec{\nabla}\cdot\vec{A}=0 \qquad \varphi=0$

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2\psi+\frac{iq\hbar}{mc}\vec{A}\cdot\vec{\nabla}\psi+\frac{q^2}{2mc^2}\vec{A}^2\psi+V(\vec{x})\psi(\vec{x})=E\psi(\vec{x})$$

$-\frac{1}{2}B(y,-x,0) = \vec{A} = -\frac{1}{2}\vec{x}\times\vec{B}$ $\qquad \vec{\nabla}\times\vec{A}=\vec{B}=B\hat{e}_z$

$-\frac{q}{2mc}\vec{L}\cdot\vec{B}=\vec{\mu}\cdot\vec{B}$

$q=-e$

$m=m_e$

$\vec{\mu}=\mu_B\frac{\vec{L}}{\hbar}$

$\mu_B=\frac{e\hbar}{2m_ec}$

پارامغناطیس

$\frac{q^2}{8mc^2}B^2\rho^2$

دیامغناطیس

$\rho^2=x^2+y^2$

$\vec{x}=(x,y,z)$

$100T=10^6G=B$

$\frac{\text{دیا}}{\text{پارا}}=\underbrace{1.1\times10^{-10}}_{G^{-1}}B \rightarrow 10^6G \sim 10^{-4}$

• $10^{18}-10^{20}\ G$

• اثر زیمان (اثر نرمال زیمان)

$$H_0=-\frac{\hbar^2}{2m_e}\vec{\nabla}^2-\frac{Ze^2}{r}$$

$Z=1$

$$H_0|nlm\rangle=E_n|nlm\rangle$$

$m=m_l$

$$E_n=-R_y\frac{1}{n^2}$$

$R_y=13.6\,eV$

$n=1,2,\dots$

$l=0,1,\dots n-1$

$-l\le m\le l \qquad 2l+1$

$H=H_0+H_{magn}$

$H_{magn}=\frac{eB}{2m_ec}L_z$

$\vec{B}\cdot\vec{L}=B\hat{e}_z\cdot\vec{L}=BL_z$

$q=-e$

m_e

$[L_z,H]=0$

$$L_z|nlm\rangle=\hbar m|nlm\rangle$$

$$H|nlm\rangle=(H_0+H_{magn})|nlm\rangle=\ \overset{?}{E}\ |nlm\rangle$$

E_n

$$H_{magn}|nlm\rangle=\frac{eB}{2m_ec}L_z|nlm\rangle=\frac{eB\hbar}{2m_ec}m|nlm\rangle$$

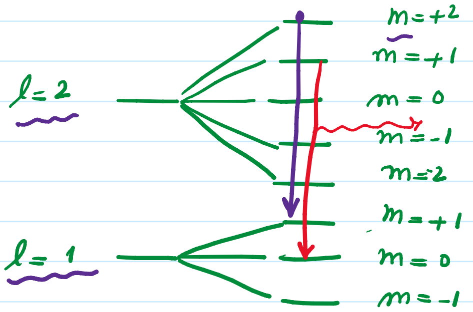
$$E_{n,m}=E_n+\hbar\omega_L m \qquad \omega_L=\frac{eB}{2m_ec}$$

فرکانس سیکلوترون

$$\Delta E_{n,m}=E_{n,m+1}-E_{n,m}=\hbar\omega_L$$

$E_{n,m} = E_n + \hbar\omega_L m$ $\qquad \omega_L = \frac{eB}{2m_e c}$ فرکانس سیکلوترون

$\Delta E_{n,m} = E_{n,m+1} - E_{n,m} = \hbar\omega_L$



$m = 0, \pm 1, \pm 2$

$m = 0, \pm 1$

selection rules

$\Delta l = 1$

$\Delta m = 0, \pm 1$

① $(l=2, m=2) \to (l=1, m=1)$

② $(2, 1) \to (1, 0)$

$(2, 0) \to (1, -1)$

$n=3 \quad l=0$ ——— $m=0$

• $\dfrac{\text{پارا}}{\text{پتانسیل کولنی}} = \dfrac{-\frac{qB}{2mc}L_z}{\frac{e^2}{r}} \sim \underbrace{2\times10^{-10}}_{G^{-1}} B \to$

$q = -e$

دیا ، پارا >> پتانسیل کولنی

$\vec{A} = -\frac{1}{2}(\vec{x}\times\vec{B})$

$= -\frac{1}{2}(y, -x, 0)B$

$$\left(-\frac{\hbar^2}{2m_e}\nabla^2 + \underbrace{\frac{eB\hbar}{2m_e c}}_{\mu_B}\frac{L_z}{\hbar} + \frac{e^2B^2}{8m_e c^2}\rho^2\right)\psi = E\psi \qquad L_z = -i\hbar\frac{\partial}{\partial\varphi}$$

دستگاه استوانه (ρ, φ, z) $\qquad x = \rho\cos\varphi,\quad y = \rho\sin\varphi,\quad z = z \qquad \rho^2 = x^2+y^2$

$$\nabla^2\psi = \left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{\partial^2}{\partial z^2}\right)\psi$$

$z \quad , \quad \varphi$ $\qquad [H, P_z] = 0$, $\quad [H, L_z] = 0$

شبه جواب $\quad \psi(\rho,\varphi,z) = e^{ikz}\, e^{im\varphi}\, u(\rho)$

$k = \frac{P_z}{\hbar}$, $\quad P_z = -i\hbar\partial_z$

$L_z = -i\hbar\partial_\varphi$, $\quad L_z|\psi\rangle = \hbar m|\psi\rangle$

$$0 = \left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \frac{m^2}{\rho^2} - \frac{e^2B^2}{4\hbar^2c^2}\rho^2\right)u(\rho) + \left(\frac{2m_eE}{\hbar^2} - \frac{eBm}{\hbar c} - k^2\right)u(\rho)$$

• تعریف دار

$$x \equiv \sqrt{\frac{eB}{2m_e c}}\,\rho$$

$$\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2} - x^2 + \lambda\right)u(x) = 0$$

$$\lambda \equiv \frac{4m_e c}{eB\hbar}\left(E - \frac{\hbar^2k^2}{2m_e}\right) - 2m$$

$x \to \infty \qquad \left(\frac{d^2}{dx^2} - x^2\right)u(x) \sim 0 \to u(x) \sim e^{-x^2/2}$

$$\lambda \equiv \frac{4m_e c}{eB\hbar}\left(E - \frac{\hbar^2 k^2}{2m_e}\right) - 2m$$

$x \to \infty$ $\quad \left(\frac{d^2}{dx^2} - x^2\right) u(x) \approx 0 \quad \longrightarrow \quad u(x) \sim e^{-x^2/2}$

$x \to 0$ $\quad \left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2}\right) u(x) \approx 0 \quad \longrightarrow \quad u(x) \simeq x^{|m|}$

$$u(x) = e^{-x^2/2}\, x^{|m|}\, G(x)$$

$x^2 = y$

$$(L_r^s)''(y) + \left(\frac{s+1}{y} - 1\right)(L_r^s)' + \left(\frac{r-s}{y}\right) L_r^s(y) = 0$$

$G(y) = L_r^s(y)$

$s = |m|$ $\qquad r - s = \dfrac{\lambda - 2 - |2m|}{4}$

$n_r = r - s$

$$E_{n_r, m} = \frac{\hbar^2 k^2}{2m_e} + \frac{eB\hbar}{2m_e c}\left(2n_r + (1+|m|) + m\right)$$

$k = \frac{P_z}{\hbar}$ $\qquad \hbar\omega_L$

✓ $\vec{A} = B(0, x, 0)$ $\qquad \vec{\nabla} \times \vec{A} = \vec{B} = B\hat{e}_z$

$\vec{A}_0 = -\frac{1}{2} B(y, -x, 0)$

$\vec{A} - \vec{A}_0 = \left(\frac{y}{2}, \frac{x}{2}, 0\right) B$ $\qquad \vec{A} \to \vec{A} - \vec{\nabla}\lambda = \vec{A}'$

$= \vec{\nabla}\lambda$ $\qquad \psi \to \psi' = e^{\frac{iq}{\hbar c}\lambda}\,\psi$

$\lambda = \left(\frac{xyB}{2}\right)$

$\hat{H}\,\psi(x,y,z) = E\,\psi(x,y,z)$

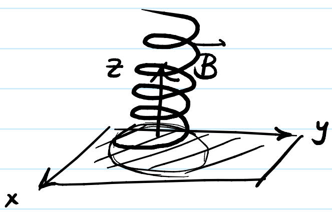
$\hat{H} = \frac{1}{2m_e}\left(\vec{P} + \frac{e}{c}\vec{A}\right)^2$ $\qquad P_x = \frac{\hbar}{i}\frac{d}{dx}$

$$= \frac{1}{2m_e}\left(-\hbar^2\frac{d^2}{dx^2} + \left(\frac{\hbar}{i}\frac{d}{dy} + \frac{eB}{c}x\right)^2 - \hbar^2\frac{d^2}{dz^2}\right)$$

$[H, P_y] = 0$ $\qquad e^{\frac{iP_y}{\hbar}y}$ $\qquad (y)$

$[H, P_z] = 0$ $\qquad e^{iP_z/\hbar\, z}$ $\qquad (z)$

$P_y = \frac{\hbar}{i}\frac{d}{dy}$ $\qquad P_z = \frac{\hbar}{i}\frac{d}{dz}$



$\psi(x,y) = e^{iky}\, v(x)$ $\qquad$ تبهگن

$k \equiv \frac{P_y}{\hbar}$

$q = -e$

$$\frac{1}{2m_e}\left(-\hbar^2\frac{d^2}{dx^2} + \left(\hbar k + \frac{eB}{c}x\right)^2\right) v(x) = E\,v(x)$$

$$\left[-\frac{\hbar^2}{2m_e}\frac{d^2}{dx^2} + \left(\frac{eB}{c}\right)^2\frac{1}{2m_e}\left(x - \frac{\hbar c k}{eB}\right)^2\right] v(x) = E\,v(x)$$

$$x - \frac{\hbar c k}{eB} = x'$$

$$dx = dx' \qquad \frac{d}{dx} = \frac{d}{dx'}$$

$$\left(-\frac{\hbar^2}{2m_e}\frac{d^2}{dx'^2} + \frac{e^2B^2}{2m_e c^2} x'^2 \right) v(x') = E v(x')$$

$$\frac{1}{2} m_e \omega^2 x'^2$$

$$\frac{1}{2} m_e \omega^2 = \frac{e^2B^2}{2m_e c^2}$$

$$\omega^2 = \frac{e^2B^2}{m_e^2 c^2} \longrightarrow \omega_B = \frac{eB}{m_e c}$$

$$E_n = \hbar \omega_B \left(n + \frac{1}{2}\right)$$

Landau ترازهای لانداو

$$\left.\begin{aligned} \vec{A} &\rightarrow \vec{A}' = \vec{A} - \vec{\nabla}\lambda \\ \varphi &\rightarrow \varphi' = \varphi + \frac{1}{c}\frac{\partial}{\partial t}\lambda \end{aligned}\right\}$$

$$\vec{B} \qquad \vec{E}$$

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t) = H \psi(\vec{x}, t)$$

$$\psi \rightarrow \psi' = e^{i\alpha(\lambda)} \psi$$

$$|\psi'|^2 \qquad |\psi|^2$$