Lecture 7-QM_II (19/07/00)

Monday, October 11, 2021 2:54 PM

$$\vec{S} = (S_x, S_y, S_z) \qquad [S_i, S_j] = i\epsilon_{ijk} S_k$$

$$\vec{L} = \vec{x} \times \vec{p} \qquad [L_i, L_j] = i\epsilon_{ijk} L_k$$

$$\vec{S}^2 \, |s, m_s\rangle = \hbar^2 s(s+1) \, |s, m_s\rangle$$

$$S_z \, |s, m_s\rangle = \hbar m_s \, |s, m_s\rangle$$

ذره اسپین $\frac{1}{2}$:

$$s = \frac{1}{2} \qquad m_s = \pm\frac{1}{2}$$

$$\overset{s}{|\tfrac{1}{2}}, \overset{m_s}{+\tfrac{1}{2}}\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec{x} \times \vec{p} = \vec{x} \times \frac{\hbar}{i} \vec{\nabla}$$

$$|\tfrac{1}{2}, -\tfrac{1}{2}\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

فضاکل

$$\vec{L} \qquad U_{\delta\varphi} = e^{\frac{i}{\hbar}\delta\vec{\varphi} \cdot \vec{L}}$$

$$\vec{x} \to \vec{x}' = \vec{x} + \delta\vec{\varphi} \times \vec{x}$$

$$\langle \vec{x} | \psi \rangle = \psi(\vec{x}) \in \mathcal{L}_2(\mathbb{R}^3)$$

$$\mathbb{R}^3$$

$$U_{\delta\varphi} \, \psi(\vec{x}) = \psi(\vec{x}')$$

$$e^{\frac{i}{\hbar}\delta\vec{\varphi} \cdot \vec{S}} \, |\uparrow\rangle = |\uparrow\rangle'$$

$$\begin{pmatrix} e^{\frac{i\delta\varphi}{2}} & 0 \\ 0 & e^{-\frac{i\delta\varphi}{2}} \end{pmatrix}_{2\times 2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{2\times 1} = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}_{2\times 1}$$

$$\underset{\text{Spinor}}{|s\rangle} = \alpha_+ |\uparrow\rangle + \alpha_- |\downarrow\rangle$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma} \qquad e^{\text{Matrix}} = \sum_{n=0}^{\infty} \frac{1}{n!} (\text{Matrix})^n$$

$$|\psi\rangle = \int d^3x' \, \psi(\vec{x}') \, |\vec{x}'\rangle$$

$$\langle \vec{x} | \psi \rangle = \psi(\vec{x})$$

$$\langle \vec{x} | \psi \rangle = \int d^3x' \, \psi(\vec{x}') \underbrace{\langle \vec{x} | \vec{x}' \rangle}_{\delta(\vec{x} - \vec{x}')} = \psi(\vec{x}) \checkmark$$

$$|\Psi\rangle = \int d^3x' \left(\psi_+(\vec{x}') \, |\vec{x}'\rangle \otimes |\uparrow\rangle + \psi_-(\vec{x}') \, |\vec{x}'\rangle \otimes |\downarrow\rangle \right)$$

$$\langle \vec{x} | \vec{x}' \rangle = \delta(\vec{x} - \vec{x}')$$

$$\langle \vec{x} | \Psi \rangle = \psi_+(\vec{x}) \, |\uparrow\rangle + \psi_-(\vec{x}) \, |\downarrow\rangle$$

$$= \psi_+(\vec{x}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \psi_-(\vec{x}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \psi_+(\vec{x}) \\ \psi_-(\vec{x}) \end{pmatrix}$$

$$\langle \uparrow | \langle \vec{x} | \Psi \rangle = \psi_+(\vec{x})$$

$$\langle \downarrow | \langle \vec{x} | \Psi \rangle = \psi_-(\vec{x})$$

$$\delta\vec{\varphi} = \delta\varphi\,\hat{n}$$

الکترون در حضور میدان الکترومغناطیسی

$$H = \left(\frac{1}{2m_e}\left(\vec{P} + \frac{e}{c}\vec{A}\right)^2 - e\varphi\right) \otimes \mathbb{1}_{2\times 2} + \mathbb{1}_{\mathcal{L}_2(\mathbb{R}^3)} \otimes \left(g\mu_B \frac{\vec{B}\cdot\vec{S}}{\hbar}\right)$$

$$\vec{P} = \frac{\hbar}{i}\vec{\nabla} \qquad \mathbb{1}_{2\times2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mu_B = \frac{e\hbar}{2m_e c}$$

$$H\,\Psi(\vec{x})$$

g: $2 \sim$ ضریب ژیرومغناطیس

$$\alpha = \frac{1}{137}$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

اگر از اطلاعات بخش فضایی هامیلتونی صرف نظر کنیم

$$i\hbar\,\partial_t \underbrace{\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix}}_{|\psi(t)\rangle} = \underbrace{H_{sp.}}_{g\mu_B \frac{\vec{B}\cdot\vec{S}}{\hbar}} \underbrace{\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix}}_{|\psi(t)\rangle}$$

$$\vec{B} = B\hat{e}_3$$

$$= g\frac{\mu_B}{\hbar}\frac{\hbar}{2}B\sigma_3 = \frac{egB\hbar}{4m_e c}\sigma_3$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$i\hbar\,\partial_t \begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} = \frac{egB\hbar}{4m_e c}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix}$$

Ansatz $\begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} = e^{-i\omega t}\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$ مطلوب است ω:

$$\cancel{\hbar}\omega\, \cancel{e^{-i\omega t}} \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix} = \frac{egB\cancel{\hbar}}{4m_e c}\,\cancel{e^{-i\omega t}}\begin{pmatrix} \alpha_+ \\ -\alpha_- \end{pmatrix}$$

1) $\omega_1 = +\dfrac{egB}{4m_e c}$ $\alpha_- = 0 \Leftarrow \alpha_- = -\alpha_-$

α_+ می‌تواند هر چیزی باشد

$$\begin{pmatrix} \alpha_+ \\ 0 \end{pmatrix}$$

2) $\omega_2 = -\dfrac{egB}{4m_e c}$ $\alpha_+ = 0 \Leftarrow \alpha_+ = -\alpha_+$

α_- می‌تواند هر چیزی باشد

$\omega_2 = -\omega_1$

$$\begin{pmatrix} 0 \\ \alpha_- \end{pmatrix}$$

$$|\psi(t)\rangle = \begin{pmatrix} \psi_+(t) \\ \psi_-(t) \end{pmatrix} = e^{-i\omega_1 t}\begin{pmatrix} \alpha_+ \\ 0 \end{pmatrix} + e^{-i\omega_2 t}\begin{pmatrix} 0 \\ \alpha_- \end{pmatrix}, \quad \omega_2 = -\omega_1$$

$$|\psi(t)\rangle = \begin{pmatrix} e^{-i\omega_1 t}\alpha_+ \\ e^{+i\omega_1 t}\alpha_- \end{pmatrix} \qquad \omega_1 = \frac{egB}{4m_e c}$$

فرض: $\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$ ویژه‌حالت عملگر S_x به ازای ویژه‌مقدار $+\frac{\hbar}{2}$

سوال: این حالت چگونه به زمان تحول پیدا می‌کند؟

فرض: $\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix}$ ویژه حالت عملگر S_x به ازای ویژه مقدار $+\frac{\hbar}{2}$

سوال: این حالت چگونه با زمان تحول پیدا می‌کند؟

الف) $\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix} = ?$

$$S_x = \frac{\hbar}{2}\sigma_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$S_x \begin{pmatrix}a\\ b\end{pmatrix} = + \frac{\hbar}{2}\begin{pmatrix}a\\ b\end{pmatrix}$$

$$\cancel{\frac{\hbar}{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} = \cancel{\frac{\hbar}{2}}\begin{pmatrix}a\\ b\end{pmatrix}$$

$$\begin{pmatrix}b\\ a\end{pmatrix} = \begin{pmatrix}a\\ b\end{pmatrix} \Rightarrow b=a$$

$$\begin{pmatrix}a\\ a\end{pmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} = \begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix} = \chi_+$$

ویژه حالت عملگر S_x به ازای ویژه مقدار $+\frac{\hbar}{2}$ است.

در $t=0$

$$\langle S_x \rangle = \langle \chi_+ | S_x | \chi_+ \rangle$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}$$
$$= \frac{\hbar}{2}$$

$t=0$

$$\langle S_y \rangle = \langle \chi_+ | S_y | \chi_+ \rangle$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}$$
$$= 0$$

$$\langle S_z \rangle = \langle \chi_+ | S_z | \chi_+ \rangle$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}$$
$$= 0$$

$$\chi_+ \overset{t=0}{=} \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} \qquad \chi_+(t) = \frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega_1 t}\\ e^{+i\omega_1 t}\end{pmatrix}$$

$$\langle S_x \rangle = \langle \chi_+(t) | S_x | \chi_+(t) \rangle$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}e^{+i\omega_1 t} & e^{-i\omega_1 t}\end{pmatrix}\frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}e^{-i\omega_1 t}\\ e^{+i\omega_1 t}\end{pmatrix}$$
$$= \frac{\hbar}{4}\begin{pmatrix}e^{i\omega_1 t} & e^{-i\omega_1 t}\end{pmatrix}_{1\times 2}\begin{pmatrix}e^{i\omega_1 t}\\ e^{-i\omega_1 t}\end{pmatrix}_{2\times 1}$$
$$= \frac{\hbar}{4}\left(e^{2i\omega_1 t} + e^{-2i\omega_1 t}\right)$$
$$= \frac{\hbar}{2}\cos 2\omega_1 t \qquad t=0$$

$$\langle S_y \rangle = \langle \chi_+(t) | S_y | \chi_+(t) \rangle$$

$$\langle S_y \rangle = \langle \chi_+(t) | \; S_y \; | \chi_+(t) \rangle$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\omega_1 t} & e^{-i\omega_1 t} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} e^{-i\omega_1 t} \\ e^{i\omega_1 t} \end{pmatrix}$$

$$= \frac{\hbar}{4} \begin{pmatrix} e^{i\omega_1 t} & e^{-i\omega_1 t} \end{pmatrix} \begin{pmatrix} -i \, e^{i\omega_1 t} \\ i \, e^{-i\omega_1 t} \end{pmatrix}$$

$$= \frac{\hbar}{4} \left(-i \, e^{2i\omega_1 t} + i \, e^{-2i\omega_1 t} \right)$$

$$= \frac{\hbar}{4} (-i) \left(e^{2i\omega_1 t} - e^{-2i\omega_1 t} \right)$$

$$= \frac{\hbar}{4} (-i) \left(2i \sin 2\omega_1 t \right)$$

$$= \frac{\hbar}{2} \sin \underset{\sim}{2\omega_1 t} \qquad t=0$$

لارمور $\omega_1 = \frac{eqB}{4m_e c}$

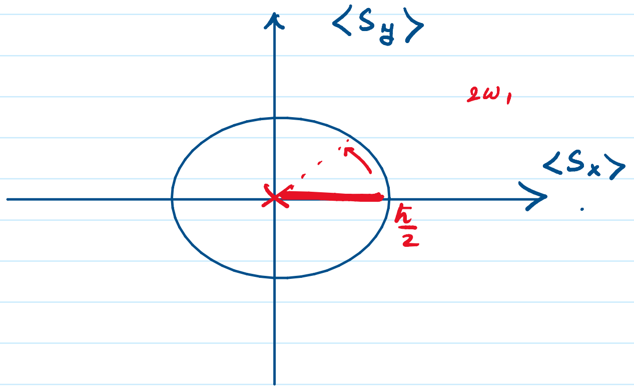
سیکلوترون $2\omega_1 = \frac{eqB}{2m_e c}$

$$\langle S_z \rangle = 0$$

$\vec{B} = B\hat{e}_z$

حرکت تقدیمی

Spin Precession



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