Lecture 8-QM_II (24/07/00)

Saturday, October 16, 2021 2:21 PM

$$\vec{S}_1 + \vec{S}_2 = \vec{S}$$

جمع تکانه زاویه‌ای :

$[S_{1i}, S_{1j}] = i\epsilon_{ijk} S_{1k}$ $\quad s_1 \quad m_{s_1}$ $\quad s \quad m_s$

$[S_{2i}, S_{2j}] = i\epsilon_{ijk} S_{2k}$ $\quad s_2 \quad m_{s_2}$

$$[\vec{S}_1, \vec{S}_2] = 0$$

$$\vec{S}_i^2 \, |s_i, m_{s_i}\rangle = \hbar^2 s_i(s_i+1)\, |s_i, m_{s_i}\rangle \qquad i = 1,2$$

ذره شماره ۱، ذره شماره ۲

$$\checkmark \; \vec{S}^2 \, |s, m_s\rangle = \hbar^2 \underline{\underline{s}}(s+1)\, |s, m_s\rangle$$

$$S_{iz} \, |s_i, m_{s_i}\rangle = \hbar m_{s_i} \, |s_i, m_{s_i}\rangle \qquad i = 1,2$$

$$\checkmark \; S_z \, |s, m_s\rangle = \hbar \underline{\underline{m_s}} \, |s, m_s\rangle$$

$\vec{S} = \vec{S}_1 + \vec{S}_2$

$$[S_i, S_j] = ? = [S_{1i} + S_{2i}, S_{1j} + S_{2j}]$$

$$= [S_{1i}, S_{1j}] + \underbrace{[S_{1i}, S_{2j}]}_{=0} + \underbrace{[S_{2i}, S_{1j}]}_{=0} + [S_{2i}, S_{2j}]$$

$$= i\epsilon_{ijk} S_{1k} + i\epsilon_{ijk} S_{2k}$$

$$= i\epsilon_{ijk}(S_{1k} + S_{2k}) = i\epsilon_{ijk} S_k$$

$$\boxed{[S_i, S_j] = i\epsilon_{ijk} S_k}$$

• فرض

$$|\uparrow\rangle_{(i)} = |s_i = \tfrac{1}{2}, m_{s_i} = +\tfrac{1}{2}\rangle$$

$$|\downarrow\rangle_{(i)} = |s_i = \tfrac{1}{2}, m_{s_i} = -\tfrac{1}{2}\rangle \qquad i = 1,2$$

$$S_{iz} \, |\uparrow\rangle_{(i)} = \frac{\hbar}{2} |\uparrow\rangle_{(i)}$$

$$S_{iz} \, |\downarrow\rangle_{(i)} = -\frac{\hbar}{2} |\downarrow\rangle_{(i)}$$

$$S_i^2 \, |\uparrow\rangle_{(i)} = \hbar^2 s_i(s_i+1) |\uparrow\rangle_{(i)} = \frac{3}{4}\hbar^2 |\uparrow\rangle_{(i)}$$

$$S_i^2 \, |\downarrow\rangle_{(i)} = \hbar^2 s_i(s_i+1) |\downarrow\rangle_{(i)} = \frac{3}{4}\hbar^2 |\downarrow\rangle_{(i)}$$

a) $|\uparrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)} = |\uparrow\uparrow\rangle$

b) $|\uparrow\rangle_{(1)} \otimes |\downarrow\rangle_{(2)} = |\uparrow\downarrow\rangle$

c) $|\downarrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)} = |\downarrow\uparrow\rangle$

d) $|\downarrow\rangle_{(1)} \otimes |\downarrow\rangle_{(2)} = |\downarrow\downarrow\rangle$

a) $S_z \, |\uparrow\uparrow\rangle = (S_{1z} + S_{2z})\left(|\uparrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)}\right)$

$$= \left(S_{1z} |\uparrow\rangle_{(1)}\right) \otimes |\uparrow\rangle_{(2)} + |\uparrow\rangle_{(1)} \otimes \left(S_{2z} |\uparrow\rangle_{(2)}\right)$$

$$= \frac{\hbar}{2} |\uparrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)} + \frac{\hbar}{2} |\uparrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)}$$

$$= \quad 1 \quad \hbar \quad |\uparrow\uparrow\rangle$$

$$= \hbar\, m_s \quad |\uparrow\uparrow\rangle \quad \Rightarrow \quad m_s = 1$$

b) $$S_z |\uparrow\downarrow\rangle = (S_{1z} + S_{2z}) \left(|\uparrow\rangle_{(1)} \otimes |\downarrow\rangle_{(2)} \right)$$

$$= \frac{\hbar}{2} |\uparrow\downarrow\rangle - \frac{\hbar}{2} |\uparrow\downarrow\rangle = 0$$

$$= \hbar\, m_s \quad |\uparrow\downarrow\rangle \quad \Rightarrow \quad m_s = 0$$

c) $$S_z |\downarrow\uparrow\rangle = (S_{1z} + S_{2z}) \left(|\downarrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)} \right)$$

$$= -\frac{\hbar}{2} |\downarrow\uparrow\rangle + \frac{\hbar}{2} |\downarrow\uparrow\rangle = 0$$

$$= \hbar\, m_s \quad |\downarrow\uparrow\rangle \quad \Rightarrow \quad m_s = 0$$

d) $$S_z |\downarrow\downarrow\rangle = (S_{1z} + S_{2z}) \left(|\downarrow\rangle_{(1)} \otimes |\downarrow\rangle_{(2)} \right)$$

$$= -\hbar \quad |\downarrow\downarrow\rangle = \hbar\, m_s \quad |\downarrow\downarrow\rangle \Rightarrow m_s = -1$$

$$\boxed{m_s = m_{s_1} + m_{s_2}}$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$+\frac{1}{2} - \frac{1}{2} = 0$$

$$-\frac{1}{2} + \frac{1}{2} = 0$$

$$-\frac{1}{2} - \frac{1}{2} = -1$$

$$S^2 \, |s, m_s\rangle = \hbar^2 \, s\,(s+1) \quad |s, m_s\rangle$$

$$\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2 = \vec{S}_1^2 + \vec{S}_2^2 + 2\vec{S}_1 \cdot \vec{S}_2 + \cancel{\underbrace{\vec{S}_2 \cdot \vec{S}_1}}$$

با درنظرگرفتن $[\vec{S}_1, \vec{S}_2] = 0$

$$= \vec{S}_1 \cdot \vec{S}_2$$

$$= \vec{S}_1^2 + \vec{S}_2^2 + 2\left(S_{1x} S_{2x} + S_{1y} S_{2y} + S_{1z} S_{2z}\right)$$

$$2\left(S_{1x} S_{2x} + S_{1y} S_{2y}\right) = S_{1+} S_{2-} + S_{1-} S_{2+}$$

$$\begin{cases} S_{ix} = \frac{1}{2}\left(S_{i+} + S_{i-}\right) \\ S_{iy} = -\frac{i}{2}\left(S_{i+} - S_{i-}\right) \end{cases}$$

$$\boxed{\vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 + 2 S_{1z} S_{2z} + S_{1+} S_{2-} + S_{1-} S_{2+}}$$

a) $$\vec{S}^2 \, |\uparrow\uparrow\rangle = \left(\vec{S}_1^2 + \vec{S}_2^2 + 2S_{1z} S_{2z} + S_{1+} S_{2-} + S_{1-} S_{2+}\right)\left(|\uparrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)}\right)$$

$$S_{1+} |\uparrow\rangle_{(1)} = 0$$

$$S_{2+} |\uparrow\rangle_{(2)} = 0$$

$$S_{i+} \, |\uparrow\rangle_{(i)} = 0$$

$$S_{i+} \, |\downarrow\rangle_{(i)} = \hbar \, |\uparrow\rangle_{(i)}$$

$$S_{i-} \, |\uparrow\rangle_{(i)} = \hbar \, |\downarrow\rangle_{(i)}$$

$$S_{i-} \, |\downarrow\rangle_{(i)} = 0$$

$$\vec{S}^2 \, |\uparrow\uparrow\rangle = \frac{3}{4}\hbar^2 \, |\uparrow\uparrow\rangle + \frac{3}{4}\hbar^2 \, |\uparrow\uparrow\rangle + 2 \, \frac{\hbar}{2} \left(\frac{\hbar}{2}\right) \, |\uparrow\uparrow\rangle + 0 + 0$$

$$= 2\hbar^2 \, |\uparrow\uparrow\rangle = \hbar^2 \, s\,(s+1) \quad |\uparrow\uparrow\rangle$$

$$s(s+1) = 2 \quad \rightarrow \quad s = 1$$

b) $S^2 \, |\uparrow\downarrow\rangle$

$$= \left(\vec{S}_1^{\,2} + \vec{S}_2^{\,2} + 2S_{1z} S_{2z} + S_{1+} S_{2-} + S_{1-} S_{2+}\right) \left(|\uparrow\rangle_{(1)} \otimes |\downarrow\rangle_{(2)}\right)$$

$$= \frac{3}{4}\hbar^2 |\uparrow\downarrow\rangle + \frac{3}{4}\hbar^2 |\uparrow\downarrow\rangle + 2\,\frac{\hbar}{2}\left(-\frac{\hbar}{2}\right) |\uparrow\downarrow\rangle + 0 + \hbar^2 |\downarrow\uparrow\rangle$$

$$S^2 |\uparrow\downarrow\rangle = \hbar^2 \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \neq \hbar^2 s(s+1)\, |\uparrow\downarrow\rangle$$

c) $S^2 \, |\downarrow\uparrow\rangle =$

$$= \left(S_1^2 + S_2^2 + 2S_{1z} S_{2z} + S_{1+} S_{2-} + S_{1-} S_{2+}\right) \left(|\downarrow\rangle_{(1)} \otimes |\uparrow\rangle_{(2)}\right)$$

$$= \frac{3}{4}\hbar^2 |\downarrow\uparrow\rangle + \frac{3}{4}\hbar^2 |\downarrow\uparrow\rangle + 2\left(-\frac{\hbar}{2}\right)\left(\frac{\hbar}{2}\right) |\downarrow\uparrow\rangle + \hbar^2 |\uparrow\downarrow\rangle + 0$$

$$= \hbar^2 \left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right)$$

d) $S^2 \, |\downarrow\downarrow\rangle$

$$= \left(S_1^2 + S_2^2 + 2S_{1z} S_{2z} + S_{1+} S_{2-} + S_{1-} S_{2+}\right) \left(|\downarrow\rangle_{(1)} \otimes |\downarrow\rangle_{(2)}\right)$$

$$= \left(\frac{3}{4}\hbar^2 + \frac{3}{4}\hbar^2 + 2\left(-\frac{\hbar}{2}\right)^2 + 0\right) |\downarrow\downarrow\rangle$$

$$= 2\hbar^2 |\downarrow\downarrow\rangle = \hbar^2 s(s+1)\, |\downarrow\downarrow\rangle \qquad s = 1$$

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$ متقارن تحت جابجایی ذره ۱ و ۲ — آری

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$ پادمتقارن تحت جابجایی ذره ۱ و ۲

$$S^2 |S\rangle = \hbar^2 s(s+1)\, |S\rangle \qquad s = 1$$

$$S^2 |A\rangle = \hbar^2 s(s+1)\, |A\rangle \qquad s = 0$$

$|S\rangle$ — جای ذره ۱ و ۲ را عوض کنیم

$$\frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right)$$

$|A\rangle$ — 〃

$$\frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle\right)$$

$$S^2\left(\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right)\right) = \frac{1}{\sqrt{2}}\left(S^2 |\uparrow\downarrow\rangle \pm S^2 |\downarrow\uparrow\rangle\right)$$

$$= \frac{\hbar^2}{\sqrt{2}}\left\{\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \pm \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)\right\} \underset{=\,0}{=} \frac{2}{\sqrt{2}}\hbar^2\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$$S^2 |S\rangle = 2\hbar^2 |S\rangle = \hbar^2 s(s+1)\, |S\rangle \Rightarrow s = 1$$

$$S^2 |A\rangle = 0 = \hbar^2 s(s+1)\, |A\rangle \Rightarrow s = 0$$

$S^2 \; |S\rangle = 2\hbar^2 \; |S\rangle = \hbar^2 s(s+1) \; |S\rangle \Rightarrow s=1$

$S^2 \; |A\rangle = 0 = \hbar^2 s(s+1) \; |A\rangle \Rightarrow s=0$

$$S_z \; \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle \right) = 0 \Rightarrow m_s = 0$$

$\vec{S}^2$	S_z		
	$m_s = 1$	$\vert\uparrow\uparrow\rangle$	سه تایی triplet
$s=1$	$m_s = 0$	$\frac{1}{\sqrt{2}} \left(\vert\uparrow\downarrow\rangle + \vert\downarrow\uparrow\rangle \right)$	
	$m_s = -1$	$\vert\downarrow\downarrow\rangle$	
$s=0$	$m_s = 0$	$\frac{1}{\sqrt{2}} \left(\vert\uparrow\downarrow\rangle - \vert\downarrow\uparrow\rangle \right)$	تک تایی Singlet

$$\vec{L} + \vec{S} = \vec{J}$$

$\vec{L}^2, \; L_z, \; \vec{S}^2, \; S_z \qquad J^2 \quad J_z$

$(l, \; m_l, \; s, \; m_s) \qquad j \quad m_j$

$$J^2 \; |j, m_j\rangle = \hbar^2 j(j+1) \; |j, m_j\rangle$$
$$J_z \; |j \; m_j\rangle = \hbar m_j \; |j, m_j\rangle$$