Lecture 9-QM_II (26/07/00)

Monday, October 18, 2021 2:51 PM

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

$$\vec{S}_i^{\,2} \, |s_i, m_{s_i}\rangle = \hbar^2 s_i (s_i+1) \, |s_i, m_{s_i}\rangle \qquad i = 1,2$$

$$S_{i_z} \, |s_i, m_{s_i}\rangle = \hbar \, m_{s_i} \, |s_i, m_{s_i}\rangle$$

$$[S_{i_1}, S_{j_1}] = i\epsilon_{ijk} S_{k_1}$$
$$[S_{i_2}, S_{j_2}] = i\epsilon_{ijk} S_{k_2}$$
$$[\vec{S}_1, \vec{S}_2] = 0$$

$$S^2 \, |s, m_s\rangle = \hbar^2 s(s+1) \, |s, m_s\rangle$$
$$S_z \, |s, m_s\rangle = \hbar \, m_s \, |s, m_s\rangle$$
$$[S_i, S_j] = i\epsilon_{ijk} S_k$$

$$\{ S_1^2, S_2^2, S_{1z}, S_{2z} \} \longrightarrow \{ S^2, S_z, S_1^2, S_2^2 \}$$

$$|s_1 \, m_{s_1}\rangle \otimes |s_2 \, m_{s_2}\rangle \longrightarrow |s, m_s; s_1, s_2\rangle$$

$$|s, m_s; s_1, s_2\rangle = \sum_{m_{s_1}, m_{s_2}} C_{m_{s_1}, m_{s_2}} \, |s_1, m_{s_1}\rangle \otimes |s_2, m_{s_2}\rangle$$

$\hookrightarrow$ Clebsh-Gordan

① $|\uparrow\rangle$ $|\downarrow\rangle$ $s_1 = \frac{1}{2}$ $m_{s_1} = \pm\frac{1}{2}$

② $|\uparrow\rangle$ $|\downarrow\rangle$ $s_2 = \frac{1}{2}$ $m_{s_2} = \pm\frac{1}{2}$

$$|s, m_s\rangle = C_1 \, |\uparrow\uparrow\rangle + C_2 \, |\downarrow\uparrow\rangle + C_3 \, |\uparrow\downarrow\rangle + C_4 \, |\downarrow\downarrow\rangle$$

S^2 $\quad S_z$

$s \quad m_s$

$|0 \; 0\rangle \qquad \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \qquad$ تکتایی Singlet

$|1, -1\rangle \qquad |\downarrow\downarrow\rangle$

$|1, 0\rangle \qquad \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \qquad$ سه‌تایی triplet

$|1, +1\rangle \qquad |\uparrow\uparrow\rangle$

$$\underbrace{} \otimes \underbrace{} \qquad 2 \otimes 2 = \underline{\underline{1}} \oplus \underline{\underline{3}}$$

a) مزون $q\bar{q}$ پایون $u\bar{u}$

تابع موج = $\cancel{\text{تابع مکانی}}$ ⊗ تابع اسپین ⊗ $\cancel{\text{تابع رنگ}}$ ⊗ $\cancel{\text{تابع طعم}}$

$\cancel{L^2}, L_z$

b) باریون $q \; q \; q$

پروتون $u u d$

نوترون $u d d$

$$\underbrace{2} \otimes \underbrace{2} \otimes \underbrace{2} =$$

$$(\vec{S}_1 + \vec{S}_2) + \vec{S}_3$$
$$\vec{S}_a + \vec{S}_3$$

$$2 = \begin{cases} 2s+1 \\ s = \frac{1}{2} \end{cases} = m_s \text{ تبهگنی}$$

$\vec{S}_1 + \vec{S}_2 \quad \begin{cases} S_a = 0 \\ S_a = 1 \end{cases} \qquad S_3 = \frac{1}{2}$

$\vec{S}_1 + \vec{S}_2$ $\left\{ \; S_a = 1 \right.$

$|S_a - S_3| \leqslant S \leqslant (S_a + S_3)$

1) $|0 - \frac{1}{2}| \leqslant S \leqslant (0 + \frac{1}{2})$ $S = \frac{1}{2}$ نقل می‌دیم $m_s = \pm \frac{1}{2}$

2) $1 - \frac{1}{2} = \frac{1}{2} \leqslant S \leqslant 1 + \frac{1}{2} = \frac{3}{2}$ $S = \frac{1}{2}, \frac{3}{2} \rightarrow m_s = \pm\frac{1}{2}, \pm\frac{3}{2}$

$m_s = m_{s_1} + m_{s_2} + m_{s_3}$

$$\left\{\begin{array}{ll} |\uparrow\uparrow\uparrow\rangle & |\tfrac{3}{2}, \tfrac{3}{2}\rangle \\ \frac{1}{\sqrt{3}} \rightarrow |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle & |\tfrac{3}{2}, \tfrac{1}{2}\rangle \\ |\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle & |\tfrac{3}{2}, -\tfrac{1}{2}\rangle \\ |\downarrow\downarrow\downarrow\rangle & |\tfrac{3}{2}, -\tfrac{3}{2}\rangle \end{array}\right\}$$

$$\left\{\begin{array}{ll} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\,|\uparrow\rangle & |\tfrac{1}{2}, \tfrac{1}{2}\rangle \\ (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\,|\downarrow\rangle & |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \end{array}\right\} \psi_{12}$$

$$\left\{\begin{array}{ll} |\uparrow\rangle\,(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) & |\tfrac{1}{2}, \tfrac{1}{2}\rangle \\ |\downarrow\rangle\,(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) & |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \end{array}\right\} \psi_{23}$$

$$2 \otimes 2 \otimes 2 = 2 \oplus 2 \oplus 4$$

$$\underset{2\times\frac{1}{2}+1}{\mathcal{H}_{1/2}} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} = 2\,\mathcal{H}_{1/2} + \underset{2\times\frac{3}{2}+1}{\mathcal{H}_{3/2}}$$

ضرایب نرمالیزاسیون

$$|\overset{s}{0}\;\overset{m_s}{0}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$\langle 0\,0|0\,0\rangle = 1$$

$$\langle 0\,0|0\,0\rangle = \frac{1}{2}\left(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right)\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{2}\Big(\underbrace{\langle\uparrow\downarrow|\uparrow\downarrow\rangle}_{1} - \underbrace{\cancel{\langle\uparrow\downarrow|\downarrow\uparrow\rangle}}_{0} - \underbrace{\cancel{\langle\downarrow\uparrow|\uparrow\downarrow\rangle}}_{0} + \underbrace{\langle\downarrow\uparrow|\downarrow\uparrow\rangle}_{1}\Big)$$

$= 1$

جدا دارل $\langle\uparrow\downarrow|\uparrow\downarrow\rangle = \langle\uparrow|\uparrow\rangle\langle\downarrow|\downarrow\rangle = 1$

جدا دارم $\langle\uparrow\downarrow|\downarrow\uparrow\rangle = \langle\uparrow|\downarrow\rangle\langle\downarrow|\uparrow\rangle = 0$

$(1\;\;0)\begin{pmatrix}0\\1\end{pmatrix}$

$$\vec{L} + \vec{S} = \vec{J}$$

$$\{\vec{L}^2, L_z, \vec{S}^2, S_z\} \longrightarrow \{\vec{J}^2, J_z; L^2, S^2\}$$ ترکیب

ℓ, s

$$|j, m_j\rangle = \sum C\,|\ell\;m_\ell\rangle \otimes |s = \tfrac{1}{2}, m_s = \pm\tfrac{1}{2}\rangle$$

$j \to ?$, $m_j \to ?$

$-\ell \leqslant m_\ell \leqslant \ell$

ادعا:

$$m_j = m_\ell + \tfrac{1}{2}$$
$$m_j = m_\ell - \tfrac{1}{2}$$

بشه جواب

$$m_j = m_\ell - \frac{1}{2}$$

اثبات $|j, m_j\rangle \overset{\text{شرط جواب}}{=} \alpha\, |\ell, m_\ell\rangle |\uparrow\rangle + \beta\, |\ell, m_\ell\rangle |\downarrow\rangle$

طرف چپ $J_z |j, m_j\rangle = \hbar m_j\, |j, m_j\rangle = \hbar m_j \left(\alpha\, |\ell, m_\ell\rangle |\uparrow\rangle + \beta\, |\ell\, m_\ell\rangle |\downarrow\rangle\right)$ *

طرف راست $\underbrace{J_z}_{L_z + S_z} \left(\alpha\, |\ell, m_\ell\rangle |\uparrow\rangle + \beta\, |\ell, m_\ell\rangle |\downarrow\rangle\right)$

$$= \alpha\, \hbar m_\ell\, |\ell\, m_\ell\rangle |\uparrow\rangle + \alpha \frac{\hbar}{2} |\ell\, m_\ell\rangle |\uparrow\rangle$$

$$+ \beta\, \hbar m_\ell\, |\ell, m_\ell\rangle |\uparrow\rangle + \beta \left(-\frac{\hbar}{2}\right) |\ell\, m_\ell\rangle |\downarrow\rangle$$

$$= \alpha\, \hbar \left(m_\ell + \frac{1}{2}\right) |\ell, m_\ell\rangle |\uparrow\rangle + \beta \hbar \left(m_\ell - \frac{1}{2}\right) |\ell, m_\ell\rangle |\downarrow\rangle \ *$$

$$\underbrace{m_j = m_\ell + \frac{1}{2}}_{m_\ell = m_j - \frac{1}{2}} \qquad m_j = m_\ell - \frac{1}{2}$$

• تعیین α , β و j

$(*)$ $J^2\, |j\, m_j\rangle = \hbar^2\, j(j+1)\, |j\ m_j\rangle$

$$J^2 = (\vec{L} + \vec{S})^2 = L^2 + S^2 + 2 L_z S_z + L_+ S_- + L_- S_+$$

$$L_\pm\, |\ell, m_\ell\rangle = \hbar \sqrt{\ell(\ell+1) - m_\ell (m_\ell \pm 1)}\ |\ell, m_\ell \pm 1\rangle$$

$$2\,(S_x L_x + S_y L_y) \overset{!}{=} L_+ S_- + L_- S_+ \qquad L_\pm = L_x \pm i L_y \quad S_\pm = S_x \pm i S_y$$

$S_+ |\uparrow\rangle = 0$ $\qquad$ $S_- |\uparrow\rangle = \hbar\, |\downarrow\rangle$

$S_+ |\downarrow\rangle = \hbar\, |\uparrow\rangle$ $\qquad$ $S_- |\downarrow\rangle = 0$

*طرف چپ $J^2\, |j\ m_j\rangle = \left(L^2 + S^2 + 2 L_z S_z + L_+ S_- + L_- S_+\right)\left(\alpha\, |\ell\, m_\ell\rangle |\uparrow\rangle + \beta\, |\ell, m_\ell\rangle |\downarrow\rangle\right)$

$$= \hbar^2 \left(\alpha\, \ell(\ell+1) + \frac{3}{4}\alpha + \alpha\left(m_j - \frac{1}{2}\right) + \beta \sqrt{\ell(\ell+1) - \left(m_j^2 - \frac{1}{4}\right)}\right) |\ell, \underbrace{m_j - \tfrac{1}{2}}_{m_\ell}\rangle |\uparrow\rangle$$

$$+ \hbar^2 \left(\beta\, \ell(\ell+1) + \frac{3}{4}\beta - \beta\left(m_j + \frac{1}{2}\right) + \alpha \sqrt{\ell(\ell+1) - \left(m_j^2 - \frac{1}{4}\right)}\right) |\ell, m_j + \tfrac{1}{2}\rangle |\downarrow\rangle$$

*سمت راست $= \hbar^2\, j(j+1) \left(\alpha\, |\ell, m_j - \tfrac{1}{2}\rangle |\uparrow\rangle + \beta\, |\ell, m_j + \tfrac{1}{2}\rangle |\downarrow\rangle\right)$

(A) $\alpha \left(j(j+1) - \ell(\ell+1) - \frac{3}{4} - \left(m_j - \frac{1}{2}\right)\right) = \beta \sqrt{\ell(\ell+1) - \left(m_j^2 - \frac{1}{4}\right)}$

(B) $\beta \left(j(j+1) - \ell(\ell+1) - \frac{3}{4} + \left(m_j + \frac{1}{2}\right)\right) = \alpha \sqrt{\quad \text{"}\quad}$

(A) . (B) $\rightarrow$

$$C = j(j+1) - \ell(\ell+1) - \frac{3}{4}$$

$$C^2 + C - \ell(\ell+1) = 0$$

$$C_\pm = \frac{1}{2}\left(-1 \pm \sqrt{\underbrace{1 + 4\ell(\ell+1)}_{(2\ell+1)^2}}\right)$$

$$C_+ = \frac{1}{2}(-1 + 2\ell + 1) = \ell = j(j+1) - \ell(\ell+1) - \frac{3}{4}$$

$$C_- = \frac{1}{2}(-1 - 2\ell - 1) = -\ell - 1 = j(j+1) - \ell(\ell+1) - \frac{3}{4}$$

$$j = \ell + \frac{1}{2}, \quad \checkmark$$

$j = l + \frac{1}{2}$ ✓

$j = l - \frac{1}{2}$ ✓

$l = j_1$, $j_2 = \frac{1}{2}$ $\quad j$

$|j_1 - j_2| \le j \le j_1 + j_2 \qquad l - \frac{1}{2} \le j < l + \frac{1}{2}$

- $j = l + \frac{1}{2}$ Ⓐ $j' \to \frac{\beta}{\alpha} = \sqrt{\frac{l - m_j + 1/2}{l + m_j + 1/2}}$

$\alpha_+ = \sqrt{\frac{l + m_j + 1/2}{2l+1}} \qquad \beta_+ = \sqrt{\frac{l - m_j + 1/2}{2l+1}}$

$$|j, m_j\rangle = |j = l \pm \tfrac{1}{2}, m_j = m_l \pm \tfrac{1}{2}\rangle$$

$$= \alpha_\pm \, |l, m_l = m_j - \tfrac{1}{2}\rangle |\uparrow\rangle + \beta_\pm \, |l, \overbrace{m_l + 1}^{m_j + \frac{1}{2}}\rangle |\downarrow\rangle$$

$m_l = m_j - \frac{1}{2}$

$$\alpha_\pm = \pm \beta_\mp = \pm\sqrt{\frac{l \pm m_j + 1/2}{2l+1}}$$