Sharif University of Technology - Department of Physics

Quantum Mechanics III - Fall 2019

Problem Set 1

Due Saturday 98/07/13

Problem 1 (10 pts): The motion of a free electron in a magnetic field

Read Section 8.6 of [1] (page 177-180), and derive all the equations in this section.

Problem 2 (10 pts): Aharonov-Bohm effect:

Consider the Schrödinger equation of an electron in external electric and magnetic fields

$$\left[\frac{1}{2m}\left(\frac{\hbar}{i}\boldsymbol{\nabla}-\frac{e}{c}\mathbf{A}(\mathbf{x},t)\right)^2+e\phi(\mathbf{x},t)\right]\psi(\mathbf{x},t)=i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t),$$

where $\boldsymbol{\nabla}\times\mathbf{A}=\mathbf{B}$ and $-\boldsymbol{\nabla}\phi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}=\mathbf{E}$.

(a) Show that the above Schrödinger equation is invariant under the gauge transformation

$$\mathbf{A}\to\mathbf{A}'=\mathbf{A}+\boldsymbol{\nabla}\Lambda,\qquad \phi\to\phi'=\phi-\frac{1}{c}\frac{\partial\Lambda}{\partial t},$$

if and only if $\psi(\mathbf{x},t)$ transforms as

$$\psi'(\mathbf{x},t)=\exp\left(\frac{ie}{\hbar c}\Lambda(\mathbf{x},t)\right)\psi(\mathbf{x},t).$$

Here, $\Lambda(\mathbf{x},t)$ is an arbitrary scalar function.

(b) Describe the Aharonov-Bohm effect. (*Hint:* See Section 7.5 of [1]).

References

[1] F. Schwabl, *Quantum Mechanics*, Fourth Edition, Springer Verlag, 2007.