Sharif University of Technology - Department of Physics

Quantum Mechanics III - Fall 2022

Problem Set 3

Due Monday 1401/08/23 at 10:30

Problem 1 (10 pts): Magnetic Dipole Transition

a) Determine the contribution of the second term of the expansion

$$\boldsymbol{j}_{\boldsymbol{k}} = \boldsymbol{j}_0 - i \int d^3x \; \boldsymbol{j}(\boldsymbol{x})(\boldsymbol{k} \cdot \boldsymbol{x}) + \cdots$$

to the transition rate $\Gamma_{m\to n;\boldsymbol{k},\lambda}$ (see the arguments in Section 16.4.7 of [1]).

b) Derive the selection rules (16.79) of [1] [Magnetic dipole transitions couple states with the same parity].

Problem 2 (10 pts): Problem 17.2 from Schwabl QM-Book [1]

Investigate the bound states of the δ-shell potential

$$V(r) = -\lambda \frac{\hbar^2}{2m} \delta(r - a).$$

It is useful to introduce the dimensionless variables $y = r/a, \xi = ka$, and $g = \lambda a$. It turns out that there is at most one bound state for each ℓ.

(a) Determine the s-wave function. Show that a bound state exists only for $g > 1$.

(b) Show that there is at most one bound state corresponding to each ℓ.

(c) Show for general ℓ that the minimum strength of the potential for the existence of a bound state is $g = 2\ell + 1$.

References

[1] F. Schwabl, *Quantum Mechanics*, Fourth Edition, Springer Verlag, 2007.