Far From Equilibrium Initial Conditions and the Search For the QCD Critical Point

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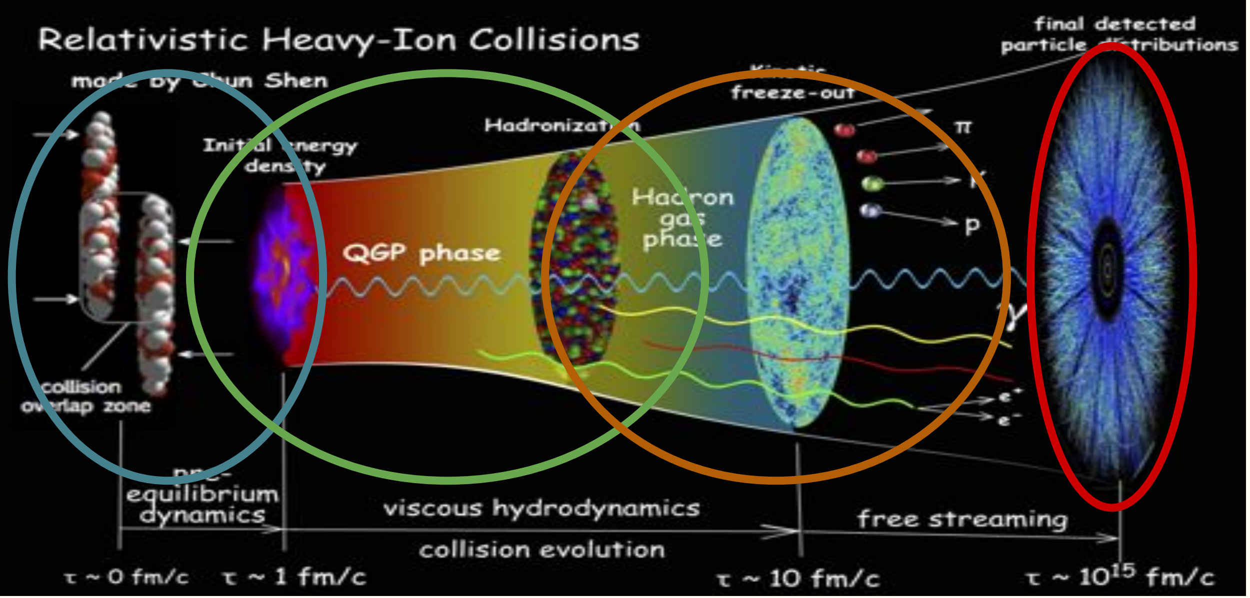


Stages of HIC

1. Initial Stages

2. Aftermath and Hydrodynamic Evolution

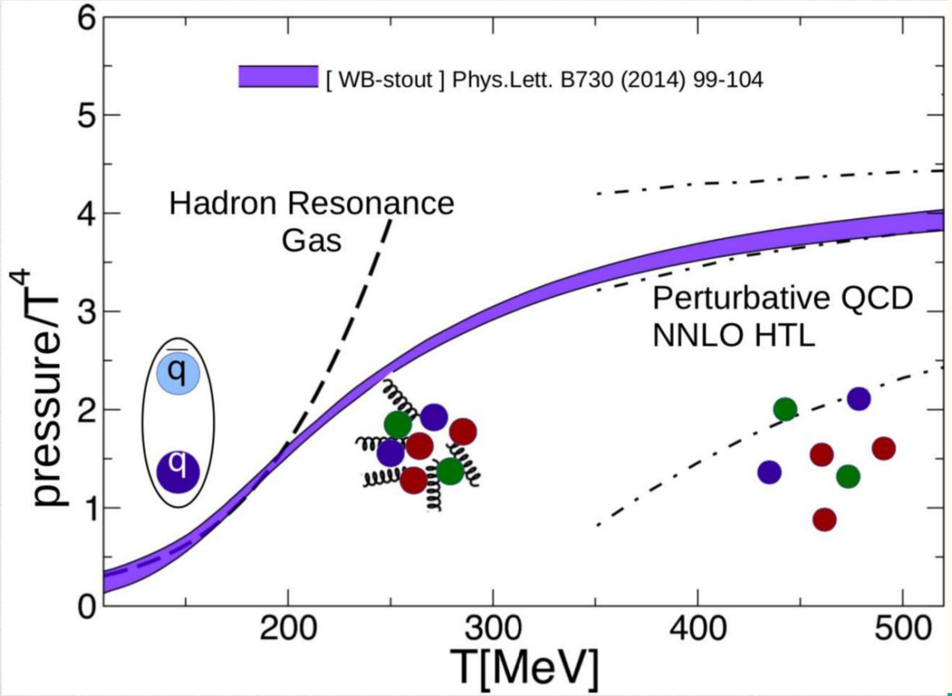
3. Hadronization, Phase Transition, Freeze Out

4. Measured Charged Particles in Momentum Space

Challenge of HIC:

Relating Complicated Theoretical Hybrid Models to Experimental Measurements

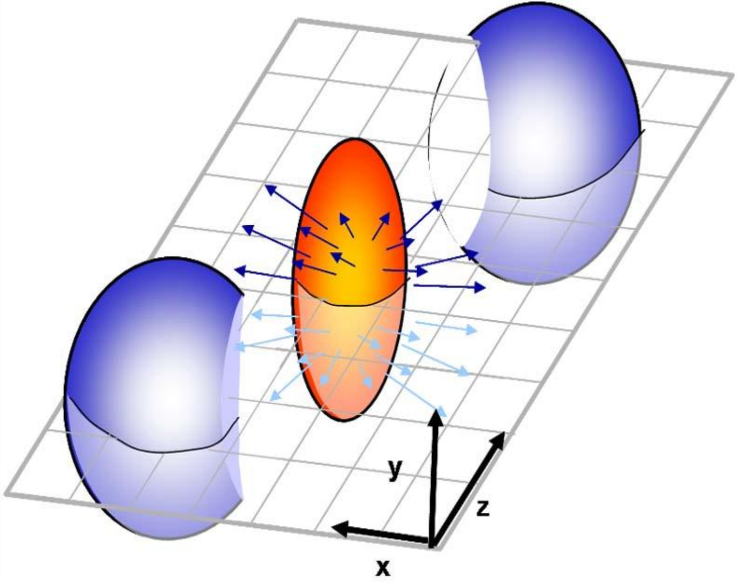
LHC and Vanishing Baryon Chemical Potential

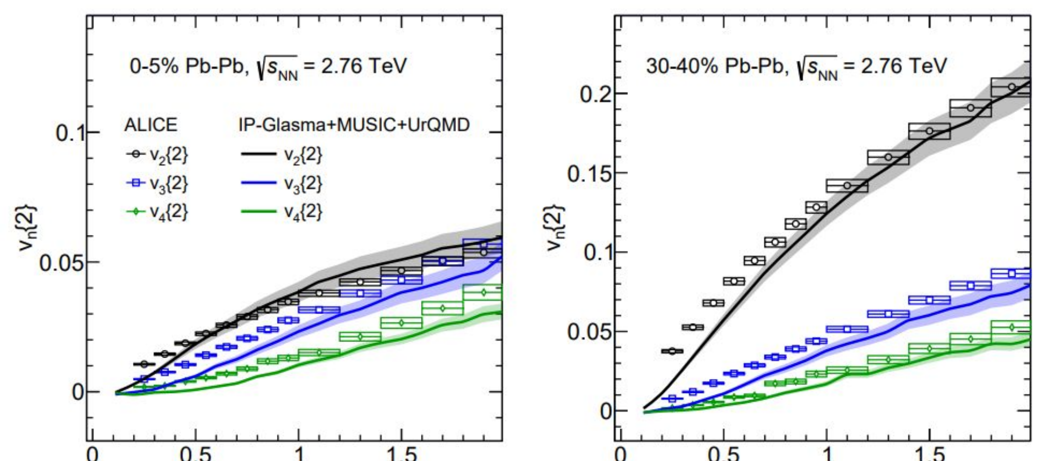


- ➢ QCD Equation of State *only* function of Temperature
- ➢ At LHC energies, nuclei punch *completely through*
- ➢ Energy deposition evolves hydrodynamically

Viscous Hydro: No Conserved Charges

- Historically has done *very well* at describing low p_T particles for a broad range of collision energies
- Many models on the market..
  - v-USPhydro, MUSIC, iEbE-Vishnu





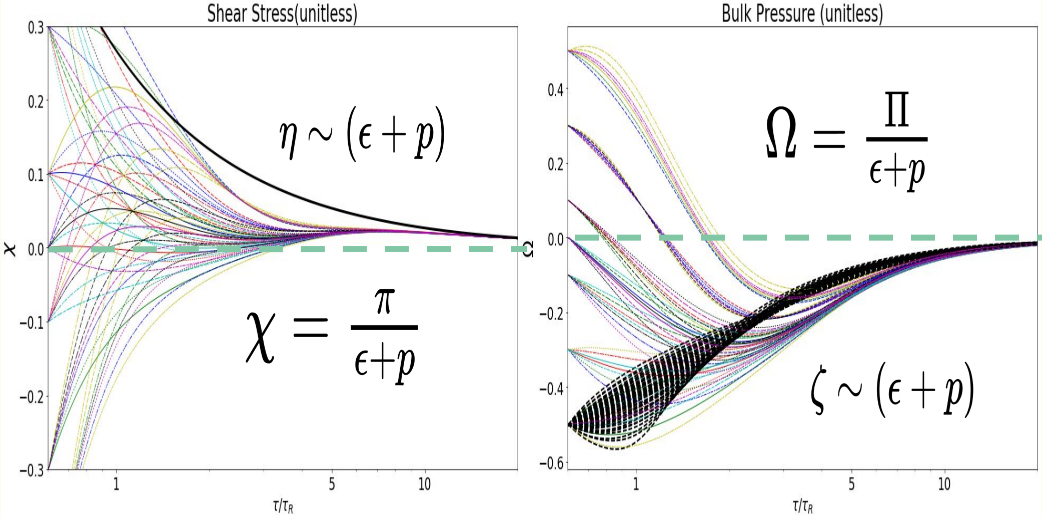
A.Dubla, et. al. *Nucl.Phys.A* 979 (2018) 251-264

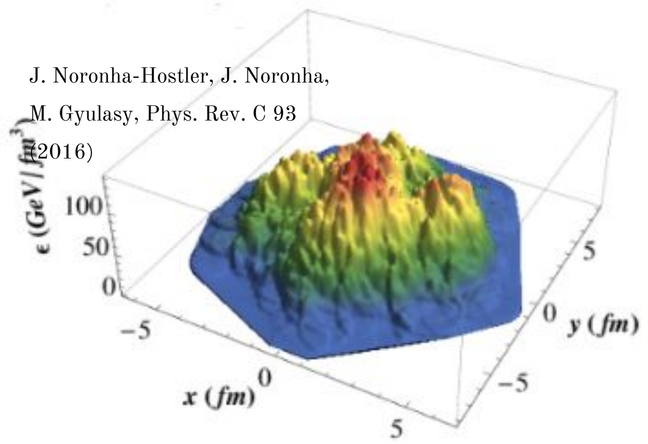
Current Efforts to Constrain Initial Conditions are Ongoing

Attractors and Far-From-Equilibrium Hydro

Seems to be *robust* feature of kinetic theory and different hydrodynamic formulations:

Heller and Spalinski, PRL 115 (2015); Gabriel S. Denicol and Jorge Noronha, Phys. Rev. D 97; Romatschke P. J., High Energ. Phys. (2017); F. Bemfica, M. Disconzi, J. Noronha Phys. Rev. D 98 (2018); M. Strickland, J. Noronha, G. Denicol, *Phys.Rev.D 97 (2018) 3*





J. Noronha-Hostler, J. Noronha, M. Gyulasy, Phys. Rev. C 93 (2016)

The existence of attractors makes FFE Hydro plausible, and even likely in small systems.

A. Bzdak, et al., *Phys.Rev.*C 87 (2013); H. Niemi, G. Denicol, arXiv:1404.7327

Attractors: a Blessing and a Curse

Attractors offer a means to explain the robust predictions of hydrodynamics given the far from equilibrium initial state of heavy ion collisions

By their very nature, attractors imply memory loss of the system's initial condition, making theoretical constraints very hard to detect

How does this look for toy model hydro with a realistic Equation of State?

Israel-Stewart vs DNMR

- ★ Independent and dynamic viscous currents that *relax* to Navier-Stokes values before equilibrium
- ★ 'Second order theory' in Knudsen and inverse Reynold's numbers

Relaxation equations with boost invariance and polar symmetry:

$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \frac{\eta T \pi^\eta_\eta}{2}\left(\beta_\pi + \tau\dot{\beta}_\pi\right)\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left[\zeta + \frac{\zeta T \Pi}{2}\left(\beta_\Pi + \tau\dot{\beta}_\Pi\right)\right]$$

W. Israel, J.M. Stewart, *Annals Phys.* 118 (1979)

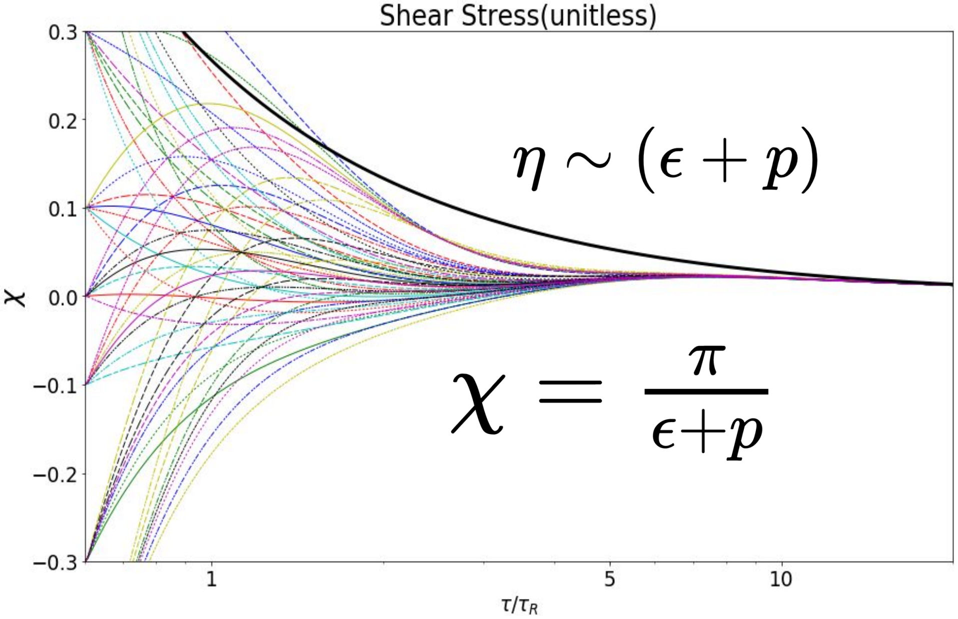
$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \pi^\eta_\eta\left(\delta_{\pi\pi} + \tau_{\pi\pi}\right) + \lambda_{\pi\Pi}\Pi\right]$$

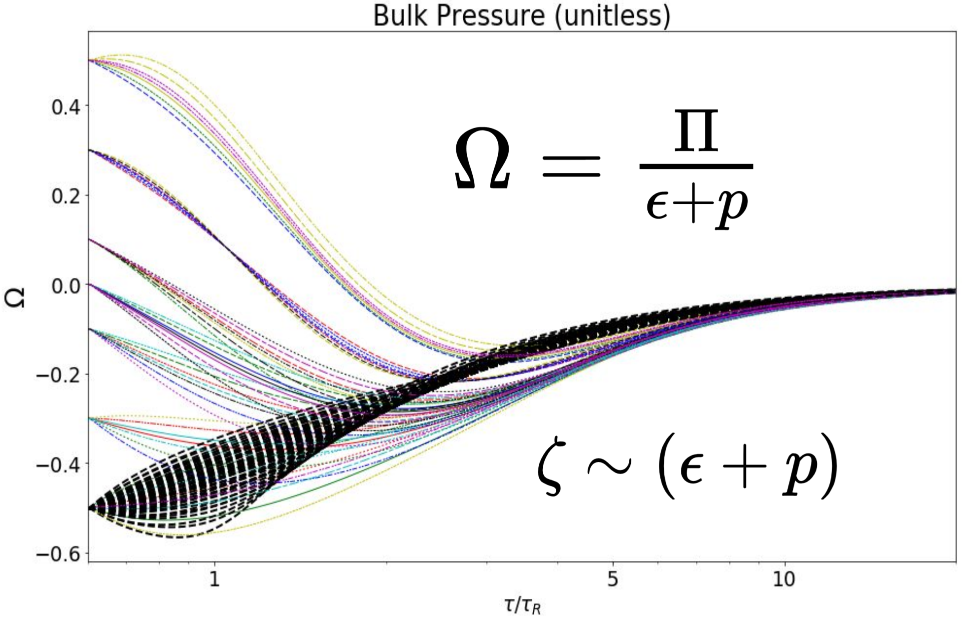
$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left(\zeta + \delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi^\eta_\eta\right)$$

G. Denicol, et al, *Eur.Phys.* J.A 48 (2012)

Biggest difference: Transport Coefficients

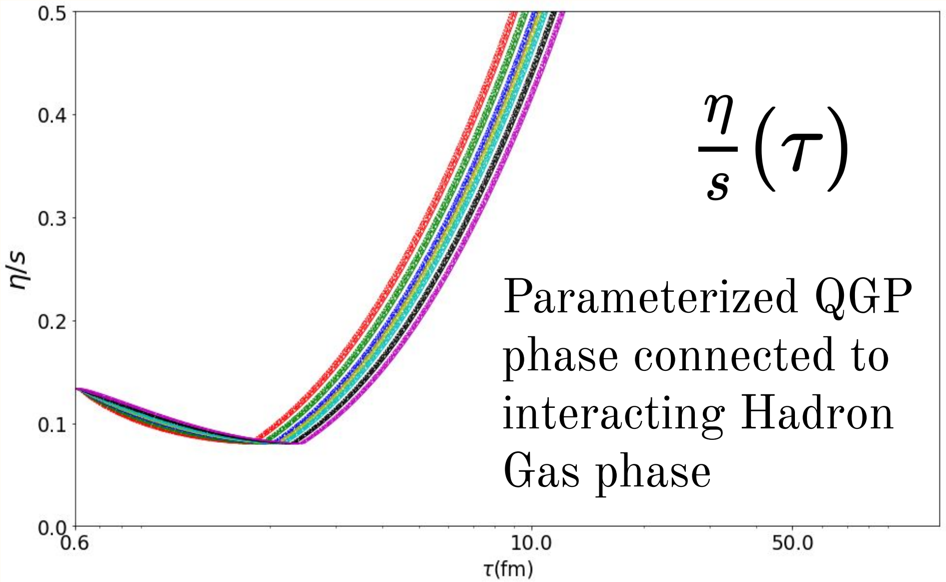
Case: Constant Relaxation Time

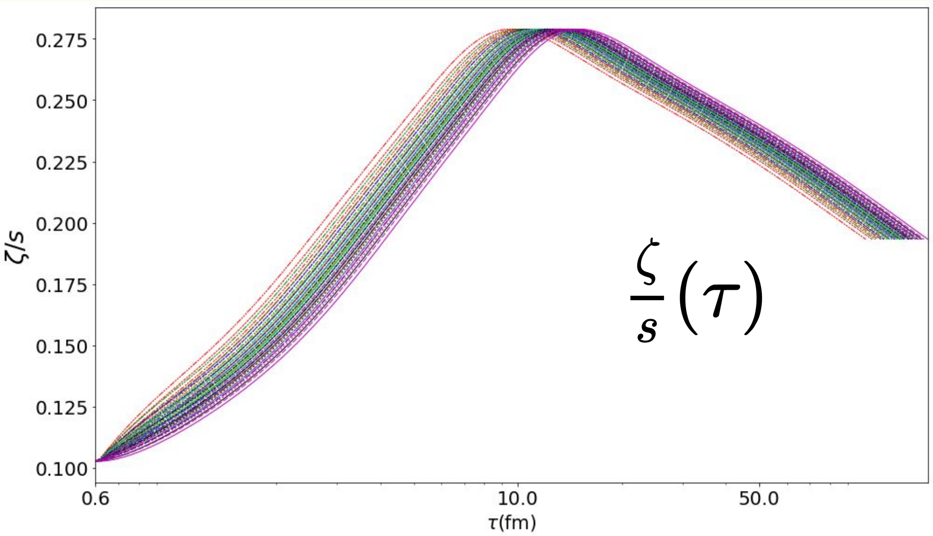




- ➢ Attracting behavior seen before Navier-Stokes (black curves)
- ➢ EoS: P. Alba, et. al, Phys Rev C98 (2018)
- ➢ System eventually reaches equilibrium

Case: Physically Motivated Transport Coefficients





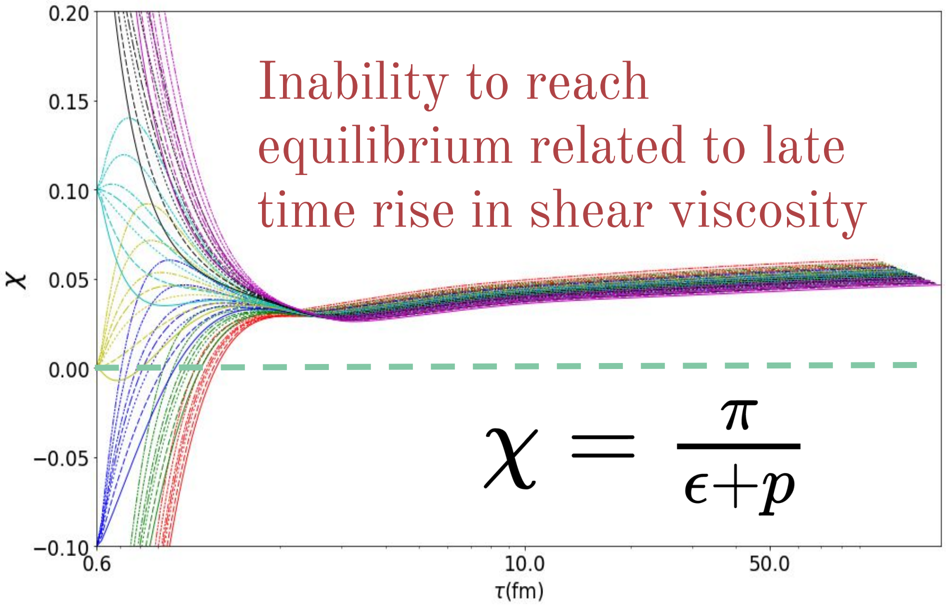
Late time rise in shear viscosity important to keep in mind

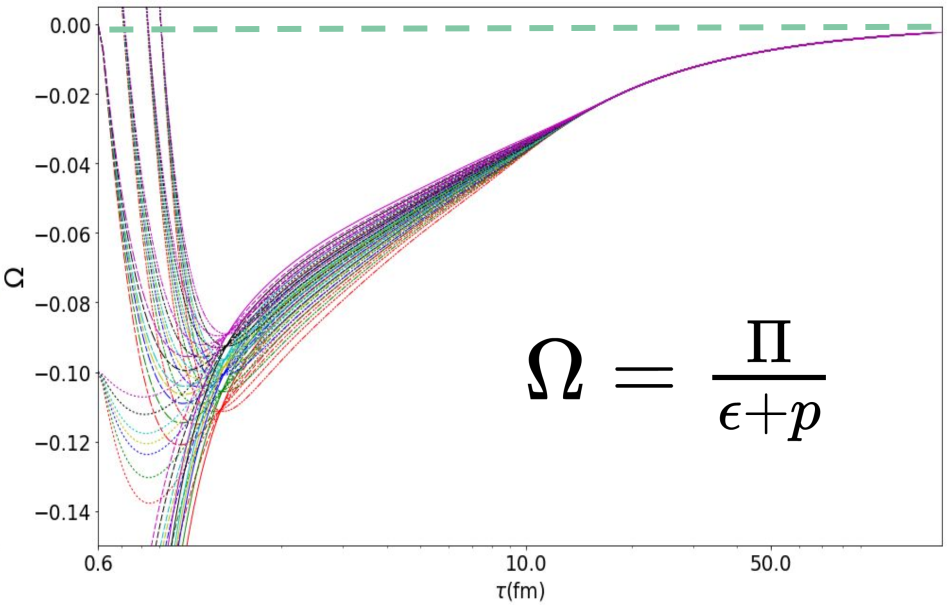
$$\frac{\zeta}{s} = 36 \times \frac{1/3 - c_s^2}{8\pi}$$

Inspired from

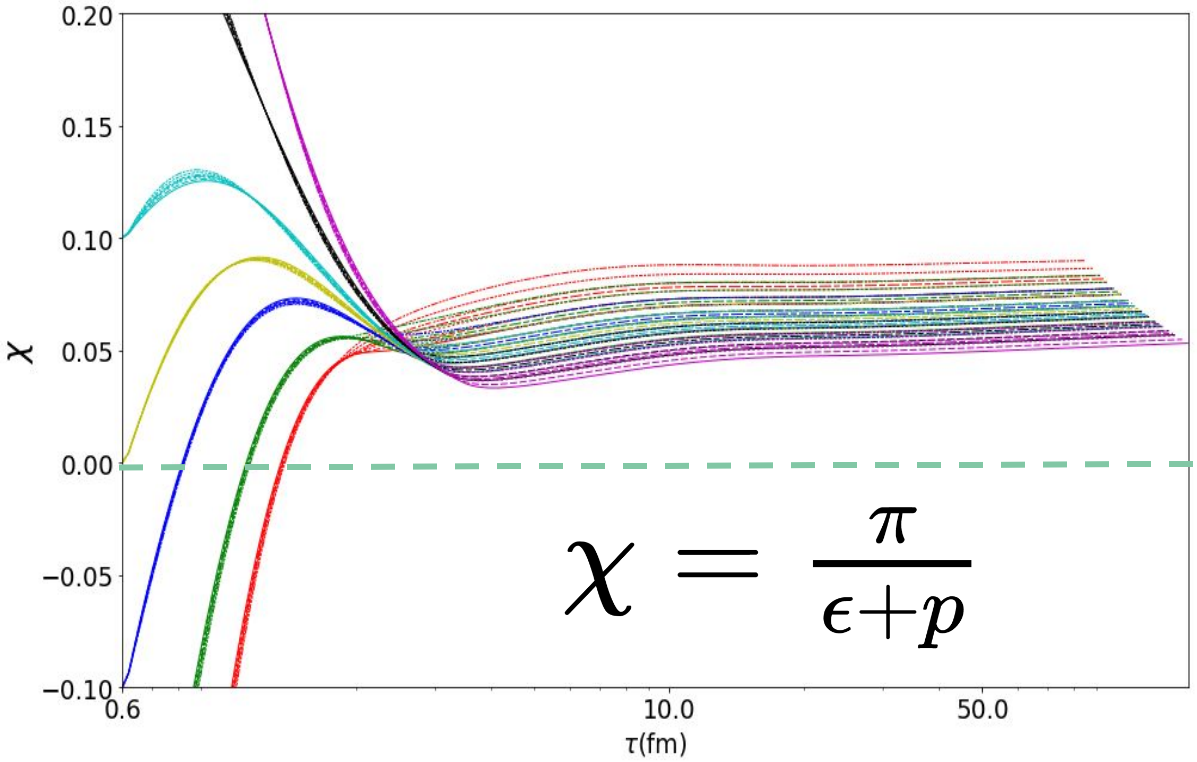
A.Buchel Lett, B663,286,2008

DNMR Admits Attracting Like Behavior





Hints that Israel-Stewart is more sensitive to initial state



$$\chi = \frac{\pi}{\epsilon + p}$$

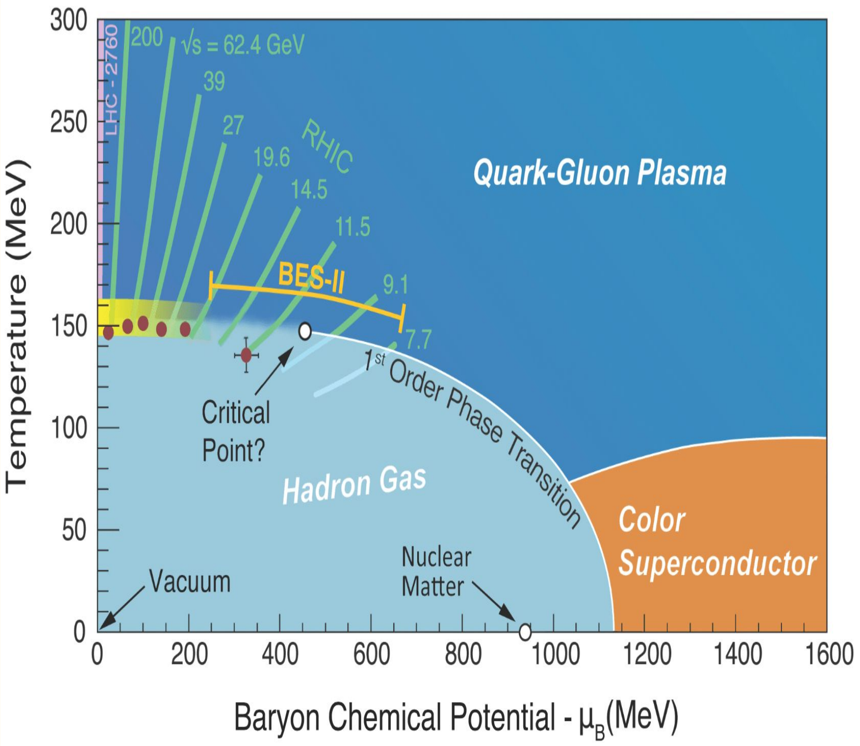
Point of Emphasis: Simplistic Study

Hints that hydro may be sensitive to initial state if the system fails to equilibrate before hadronization

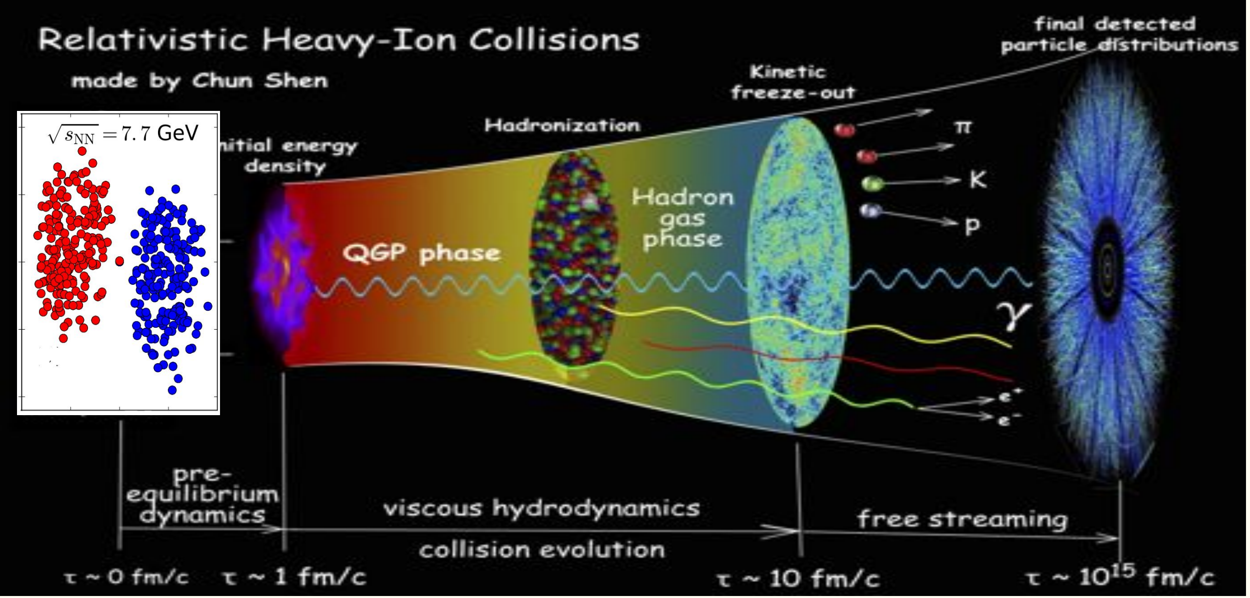
Studies of this nature must be done in more realistic scenarios to help put constraints on initial state

How far can we push this simplistic model?
What changes when including conserved charges?

The Beam Energy Scan (BES)



- Main Objective:
  Probe thermal behavior of QCD in Baryon rich regime
- Onset of criticality?
- Complicated by lower energies, finite lifetime, out of equilibrium effects



Initial State

- Baryon Stopping (some work has been done) C. Shen, B. Schenke *Phys.Rev.* C. **97**
- Initializing full $T^{\mu\nu}$
  - Also problem at $\mu_B = 0$

Equation of State

- QCD EoS and Fermi Sign Problem

Freeze-out

- Need to conserve locally, not just on average D. Oliinychenko, et al., *Phys. Rev.C* 102 (2020) 3
- Out of equilibrium corrections
  - Also problem at $\mu_B = 0$

Hydro Implementation

- (3+1)D with finite BSQ
- Transport Coefficients?
- Critical Fluctuations
- Correct Formulation?

The rest of this talk touches on a few different areas of needed research

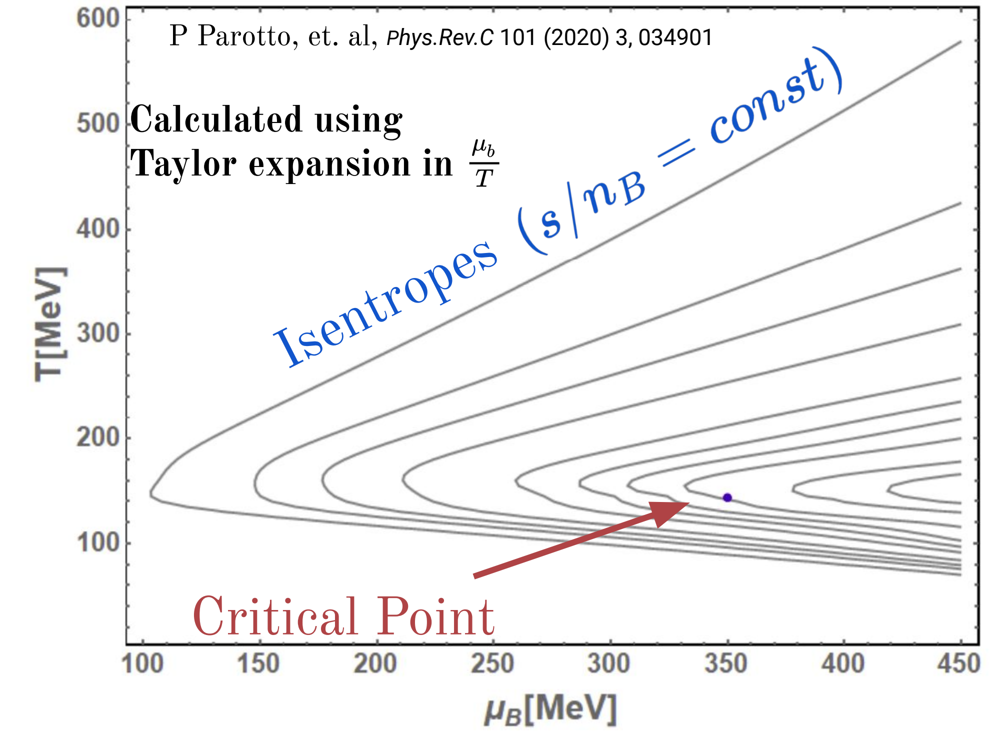
Phenomenological Tool:

Lattice QCD EoS with Parameterized CP (3D Ising University Class)

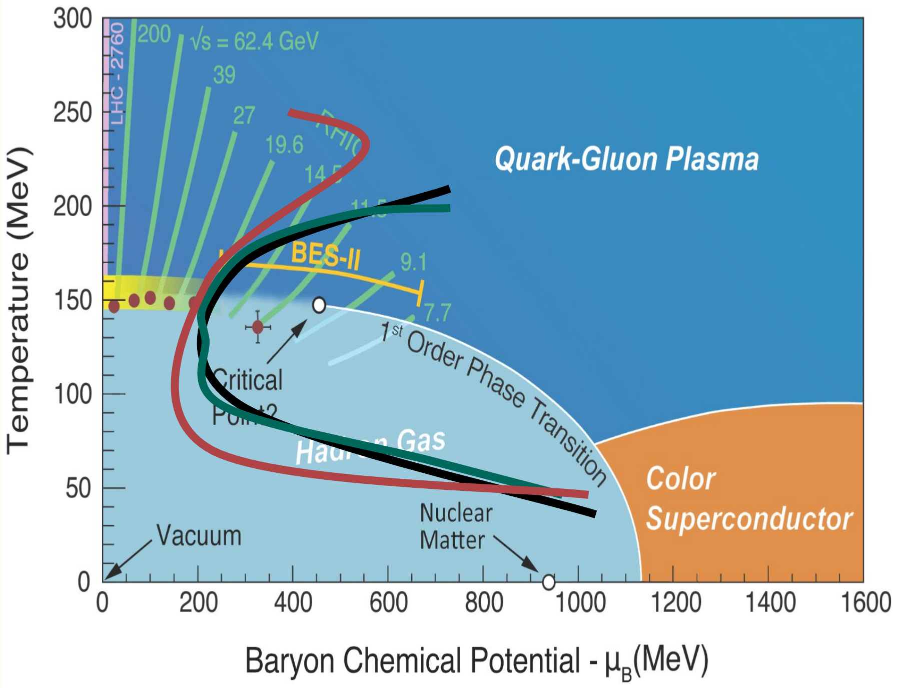
- Ideal hydrodynamics evolves along isentropic trajectories

- How do out of equilibrium effects influence trajectories?

- Effect of criticality?

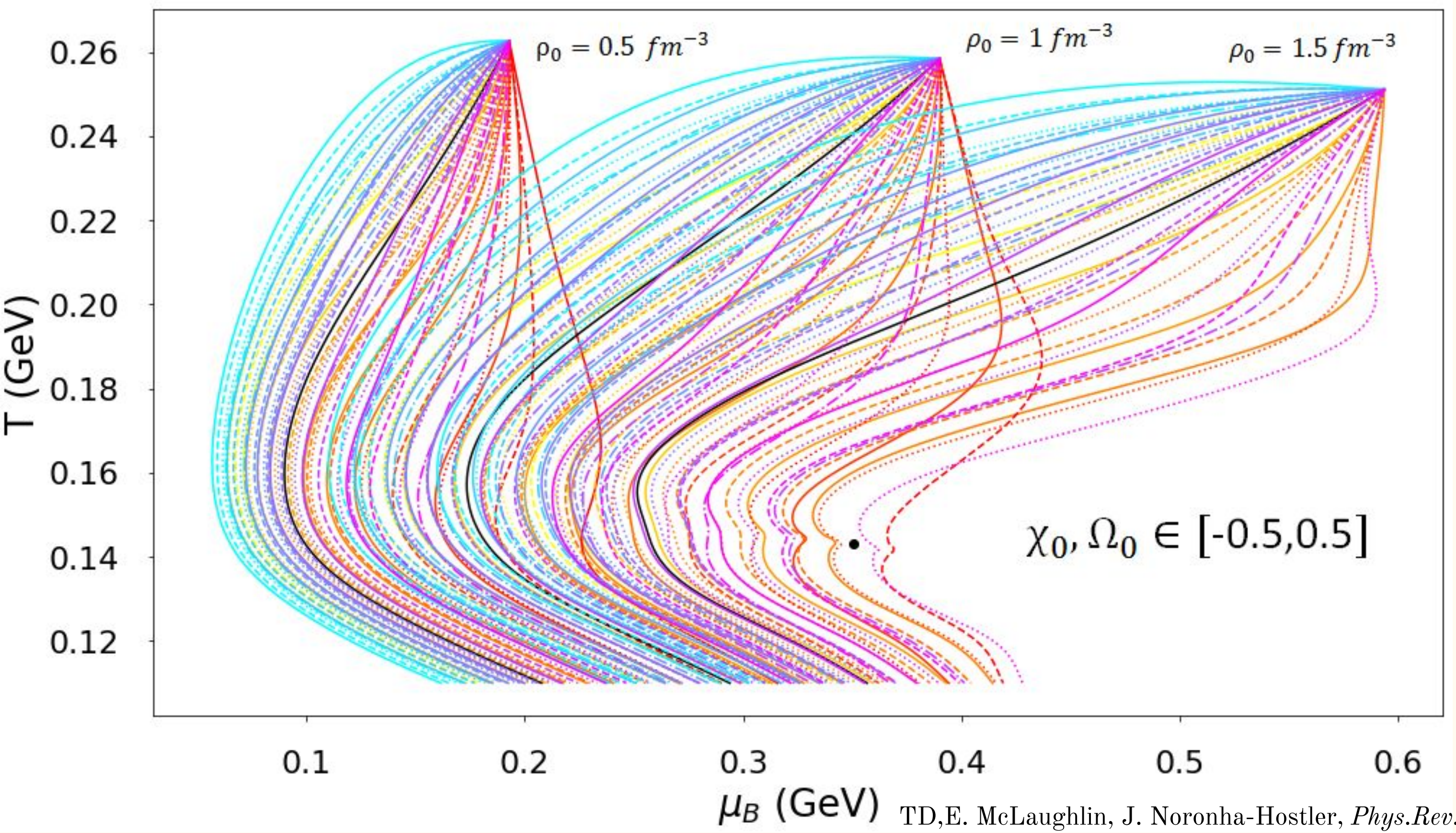


An Evolving Hydrodynamic Picture



- Without viscous effects there is no entropy production. System evolves isentropically
- Close to equilibrium scenario may not alter trajectories dramatically
- Given FFE initial conditions, changes may be significant. In fact..

Out Of Equilibrium Effects Are Important



TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

Pushing a Simple Model Further

Same hydro equations, but now we also solve: $\dot{\rho} = -\frac{\rho_0}{\tau} \Rightarrow \rho(\tau) = \frac{\rho_0 \tau_0}{\tau}$

- ➢ Bjorken symmetric flow has no charge diffusion

- ➢ Non-trivial path in phase diagram driven by non-trivial dependence of $\epsilon(\tau)$ as well as mapping $\{\epsilon, \rho\} \rightarrow \{T, \mu_B\}$

$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \frac{\eta T \pi^\eta_\eta}{2}\left(\beta_\pi + \tau\dot{\beta}_\pi\right)\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left[\zeta + \frac{\zeta T \Pi}{2}\left(\beta_\Pi + \tau\dot{\beta}_\Pi\right)\right]$$

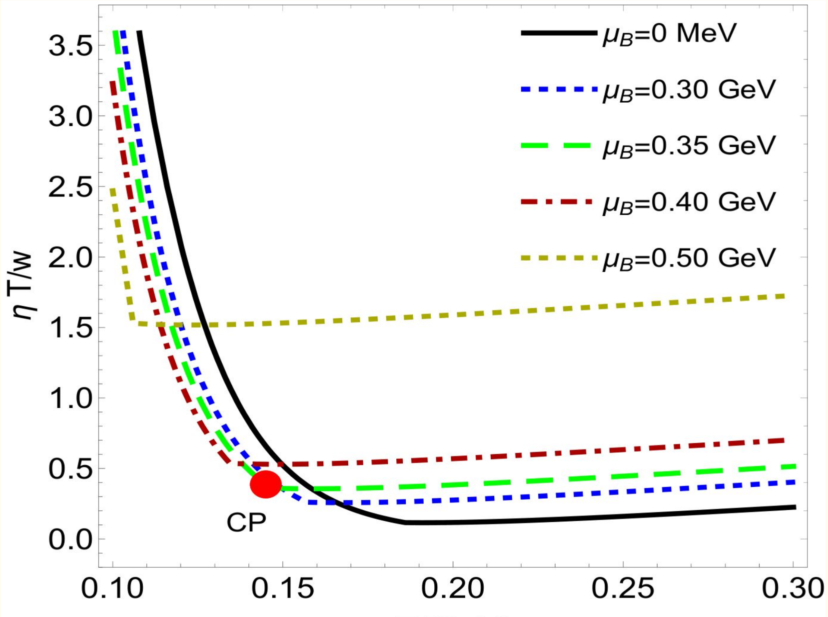
W. Israel, J.M. Stewart, *Annals Phys.* 118 (1979)

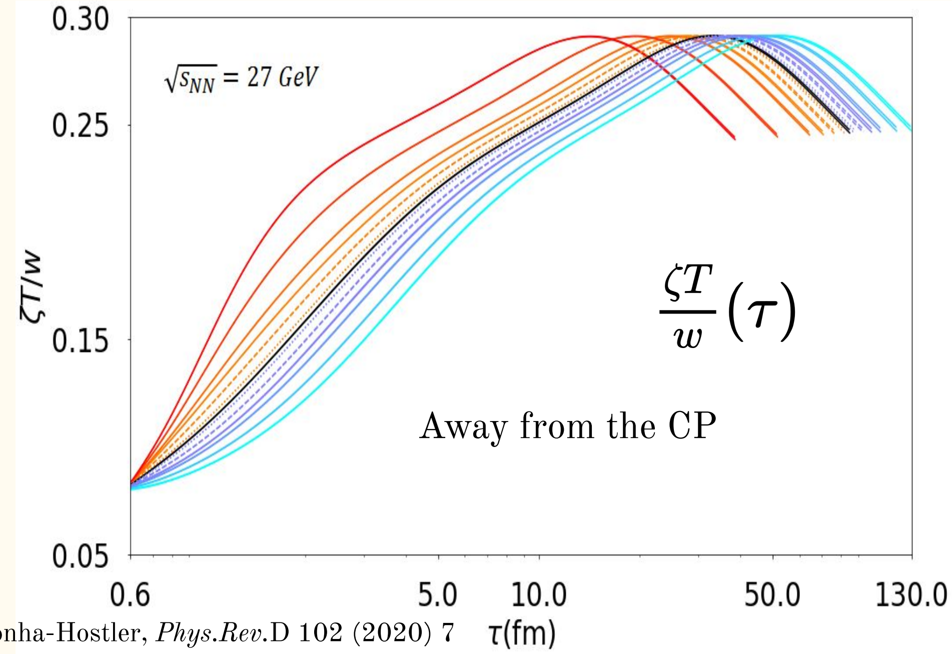
$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \pi^\eta_\eta\left(\delta_{\pi\pi} + \tau_{\pi\pi}\right) + \lambda_{\pi\Pi}\Pi\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left(\zeta + \delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi^\eta_\eta\right)$$

G. Denicol, et al, *Eur.Phys.* J.A 48 (2012)

Similar Transport Coefficients To Previous Study





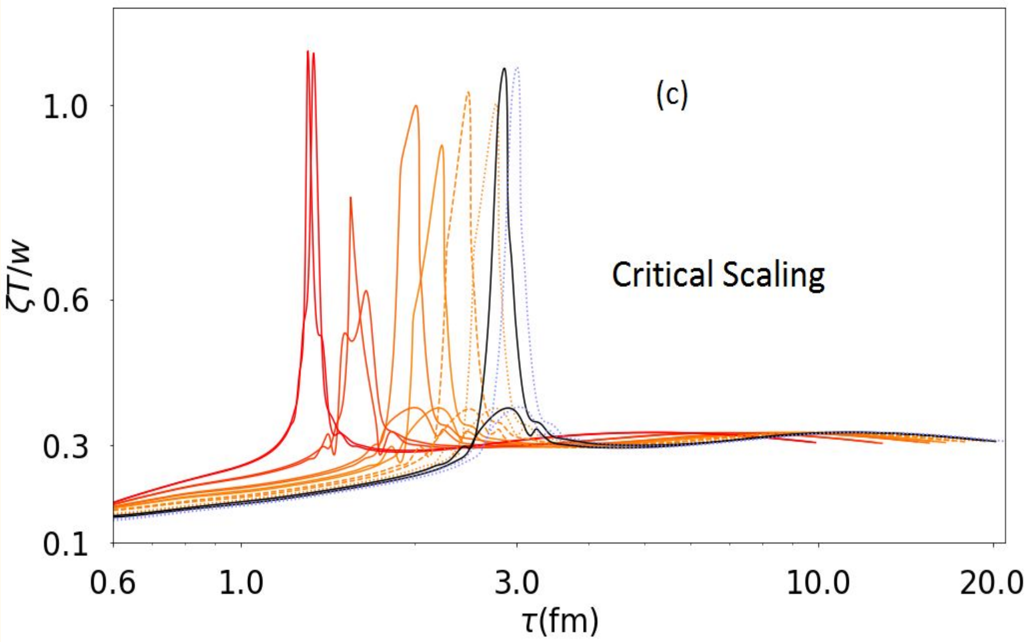
TD, E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

Shear viscosity not sensitive to criticality explicitly

Bulk viscosity away from CP shows similar behavior

Influence of Criticality on Bulk Viscosity

TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7



Monnai, Akihiko et al, Nucl. Phys. ,A967,2017

Critically Scaled Bulk:

$$\left(\frac{\zeta T}{w}\right)_{CS} = \frac{\zeta T}{w}\left[1+\left(\frac{\xi}{\xi_0}\right)^3\right]$$

Away from the critical point, only has effect from speed of sound

Correlation length calculated in linear parametrization model

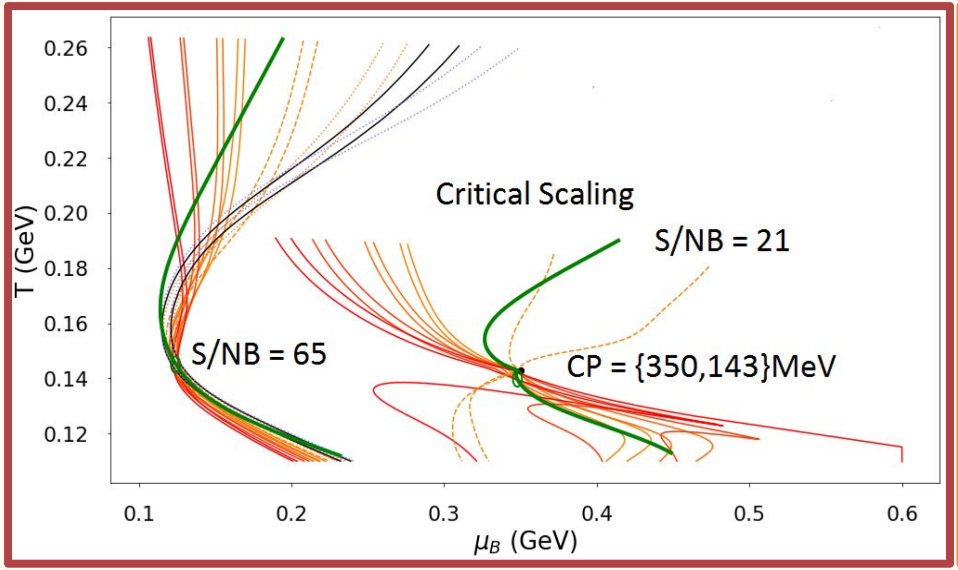
Systematic Formulation Comparison

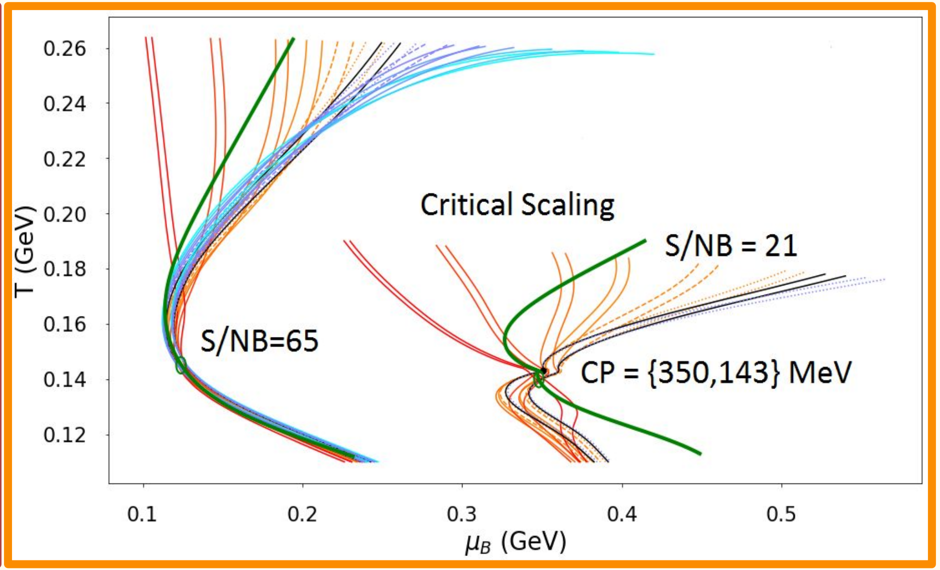
- Run *many* hydro events, systematically scan through initial conditions for $\{\chi, \Omega\}$
  - That is, the full $T^{\mu\nu}$
- Only select on events that pass through the same freeze-out point
  - Taken from:
    P. Alba, et al. *Phys.Rev.C* 101 (2020) 5
  - Ideal hydro base of comparison

Israel-Stewart

$\{T(\tau), \mu_B(\tau)\}$

DNMR





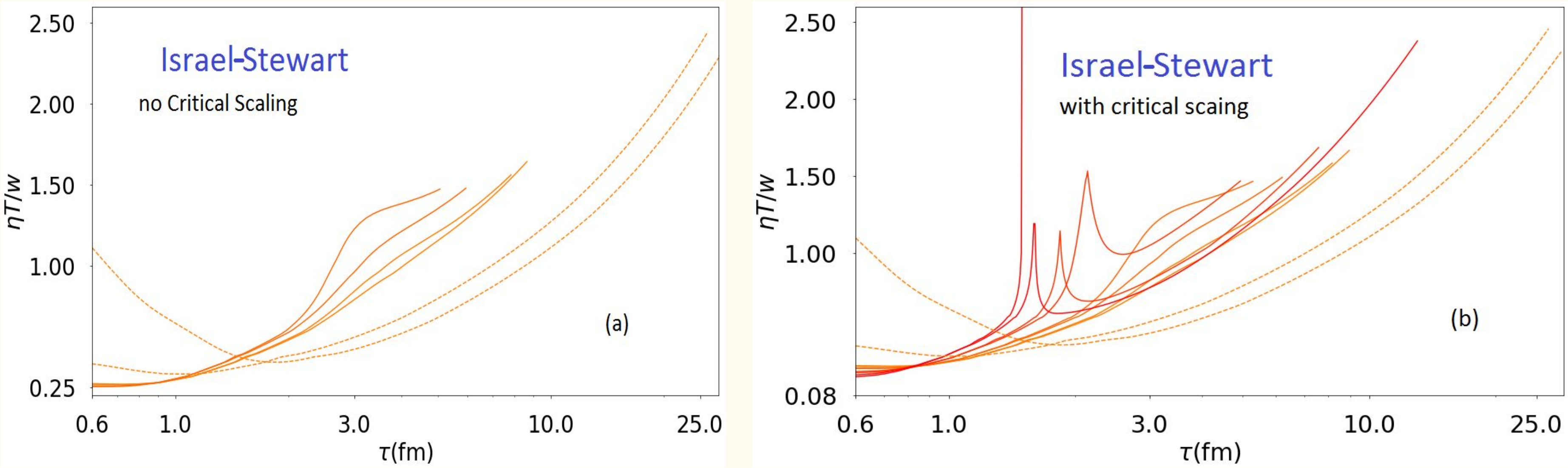
TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

Green Line: Equilibrium Hydro trajectory

Takeaways:

1. Pushed to or away from CP on EbE basis
2. Degeneracy of final state mapping to initial
3. DNMR seems more robust than Israel-Stewart

Implicit Critical Effects On Shear Viscosity: Israel-Stewart



Strange effects for shear viscosity related to instabilities as Israel-Stewart system traverses critical region

TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

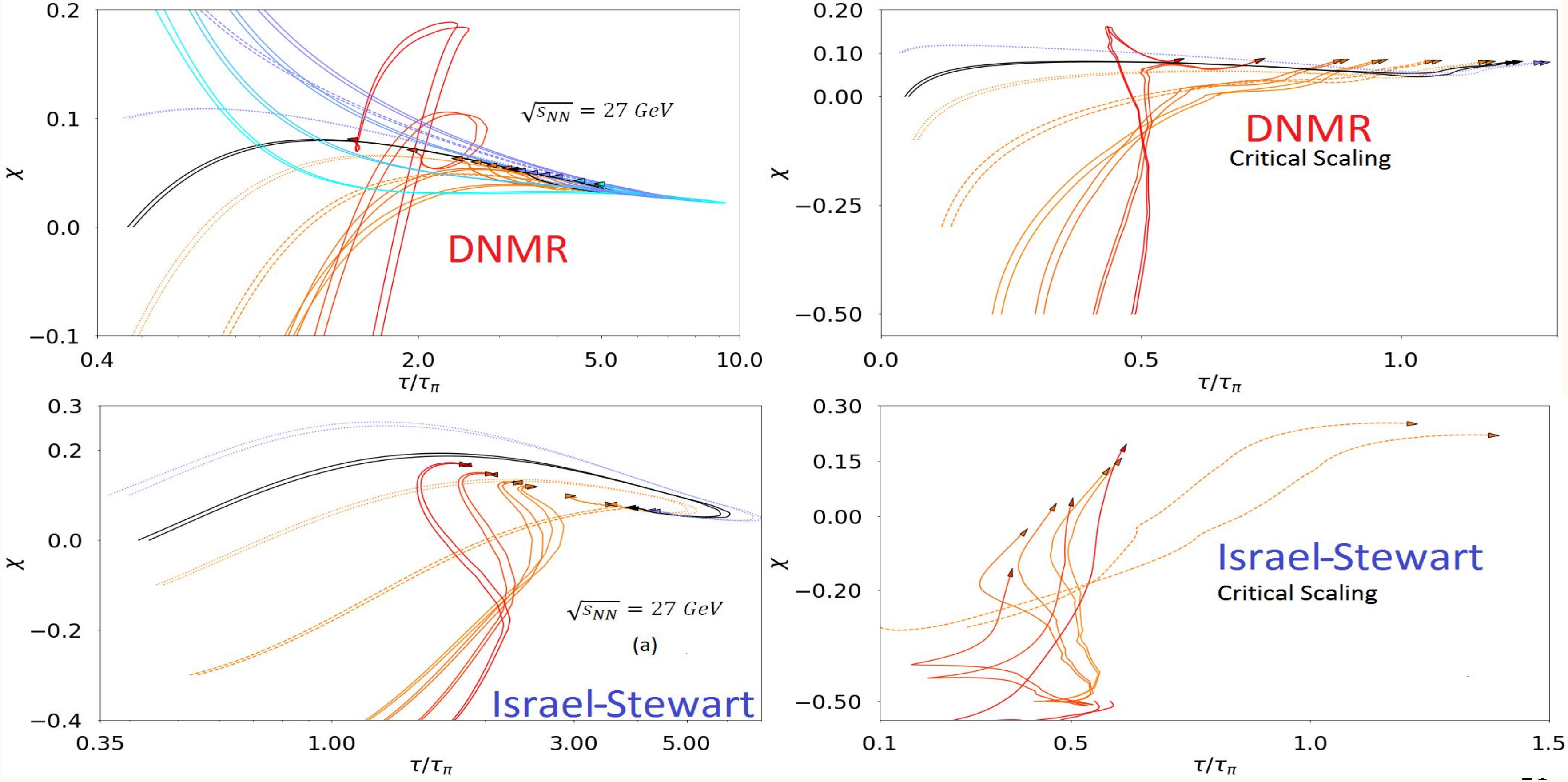
$$\chi = \frac{\pi}{\epsilon+p}$$

Far From CP

Close to CP

DNMR

Israel-Stewart



TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

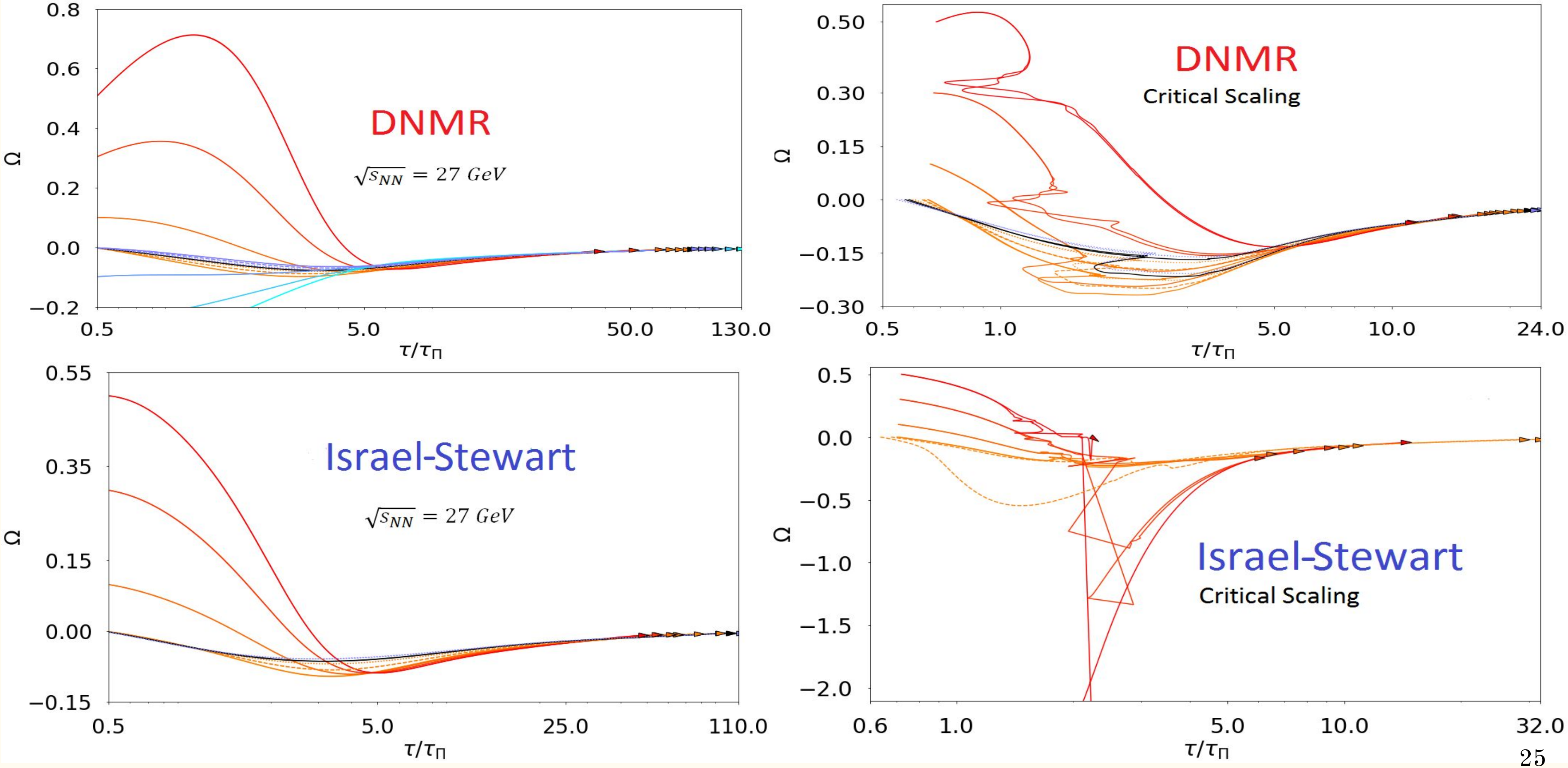
Far From CP

$$\Omega = \frac{\Pi}{\epsilon + p}$$

Close to CP

DNMR

Israel-Stewart



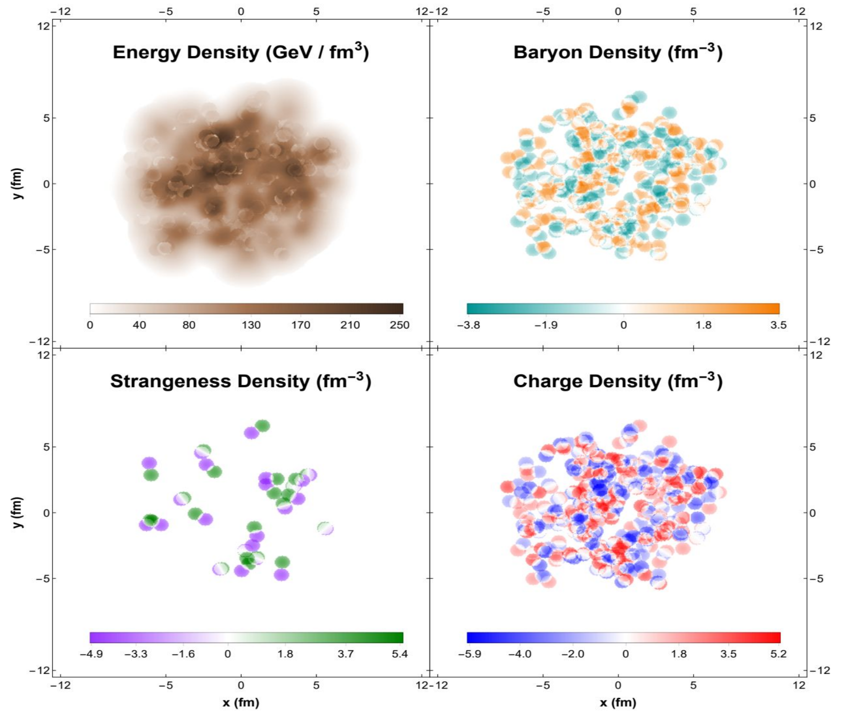
Next Steps towards BES Hydrodynamics

Start with what we know: LHC energies and ICCING

Initializing
Conserved
Charges
in Nuclear
Geometries

M. Martinez, et al., arXiv:1911.12454

M. Martinez, et al., arXiv:1911.10272



Initializing charges at 0 ***net*** density allows study of diffusion in a better controlled environment

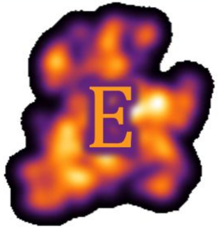
Requirements:

- (2+1) dimensional Hydro
  - V-USPhydro
- 4D EoS Noronha-Hostler, et al. *Phys.Rev.C 100 (2019)*
  - $\{\epsilon, \rho_B, \rho_Q, \rho_S\} \rightarrow \{T, \mu_B, \mu_Q, \mu_S\}$

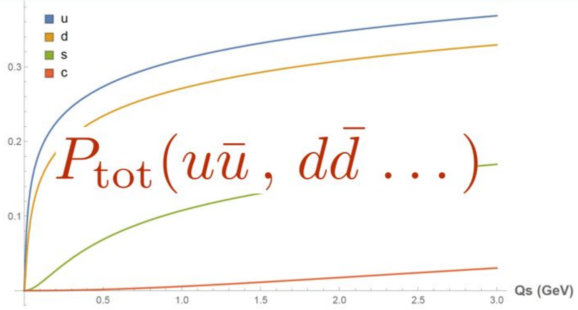
ICCING Algorithm In a Nutshell

Borrowed from talk given by
M.D. Sievert for BEST colab

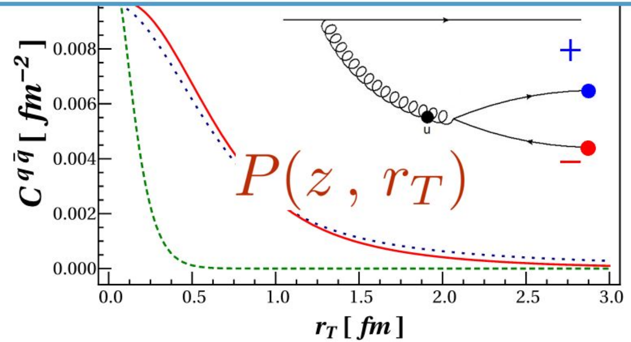
Input: Initial Energy



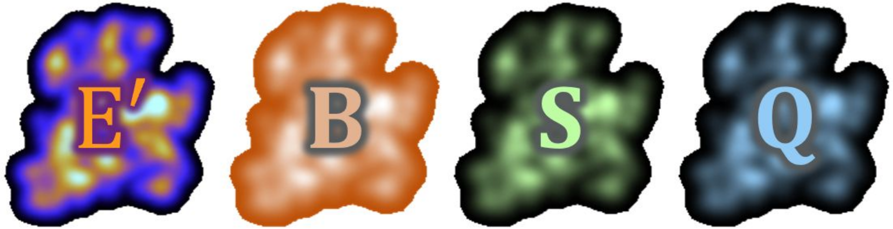
Step 1: Sample the Chemistry



Step 2: Sample the Splitting

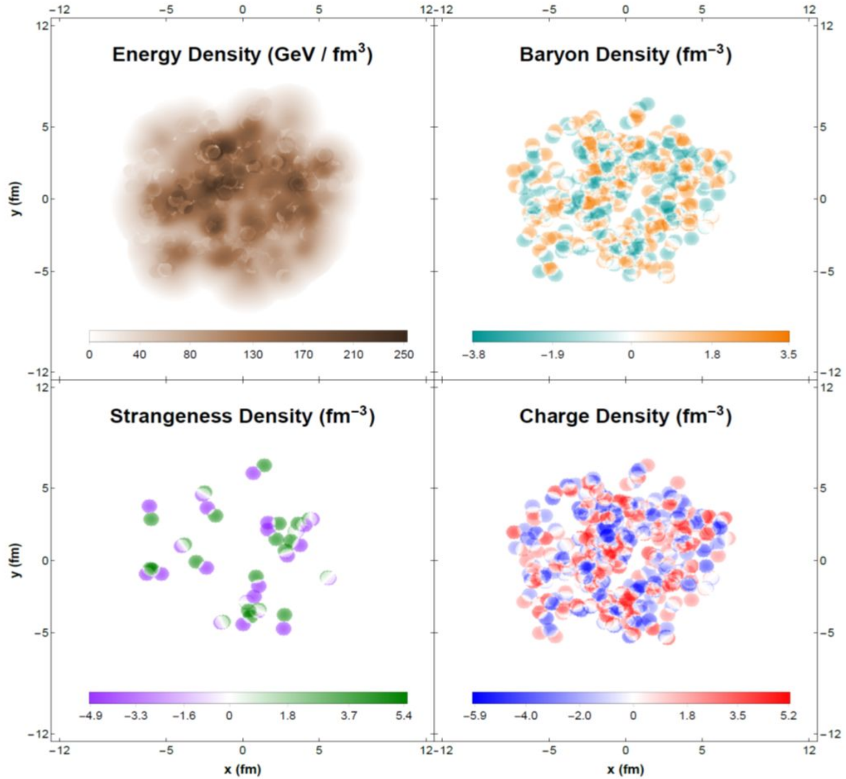


Output: Baryon #, Strangeness, Charge



ICCING Physical Picture

Borrowed from talk given by M.D. Sievert for BEST colab



Smooth Particle Hydrodynamics In a Nutshell

Two Main Approximations:

1. Coarse grain value of local quantities with kernel function

2. Represent system with finite number of SPH "*particles*", introduces notion of "*reference density*"

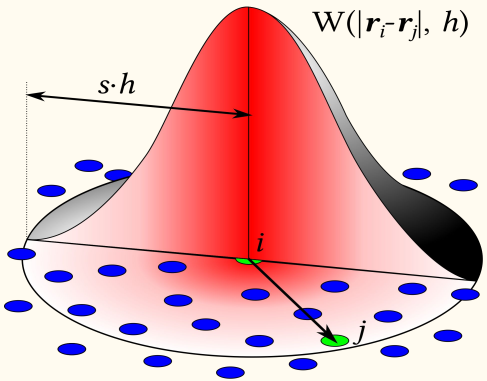
$$\int d\mathbf{r}' W(|\mathbf{r}-\mathbf{r}'|, h) = 1$$

$$with\ W(|\mathbf{r}-\mathbf{r}'|, h) \rightarrow 0\ for\ |\mathbf{r}-\mathbf{r}'| \sim \mathcal{O}(h)$$

$$\sigma = \frac{1}{V} = \frac{\gamma}{V^*}\ \ Local\ conserved\ fluid\ cell\ volume$$

Any local quantity can then be expressed as:

$$a_{SPH}(\mathbf{r},t) = \sum_{\alpha}^{N_{SPH}} \nu_\alpha \frac{a(\mathbf{r}_\alpha,t)}{\sigma^*(\mathbf{r}_\alpha,t)} W(|\mathbf{r}-\mathbf{r}'|), h)$$



Keeping Track of Reference Density: Example

$$\Pi + \tau_\Pi \frac{d}{d\tau}\Pi + = -\zeta\theta \implies \frac{\Pi}{\sigma} + \tau_\Pi \frac{d}{d\tau}\left(\frac{\Pi}{\sigma}\right) = -\frac{\zeta}{\sigma}\theta$$

$$\frac{\Pi}{\sigma} + \tau_\Pi \frac{1}{\sigma}\frac{d}{d\tau}\Pi + \tau_\Pi \Pi \frac{d}{d\tau}\frac{1}{\sigma} = -\frac{\zeta}{\sigma}\theta$$

$$\Pi + \tau_\Pi \frac{d}{d\tau}\Pi + \tau_\Pi \Pi \sigma \frac{d}{d\tau}\frac{1}{\sigma} = -\zeta\theta$$

Use relation

$$\sigma \frac{d}{d\tau}\frac{1}{\sigma} = \theta$$

$$\Pi + \tau_\Pi \frac{d}{d\tau}\Pi + = -(\zeta + \tau_\Pi \Pi)\theta$$

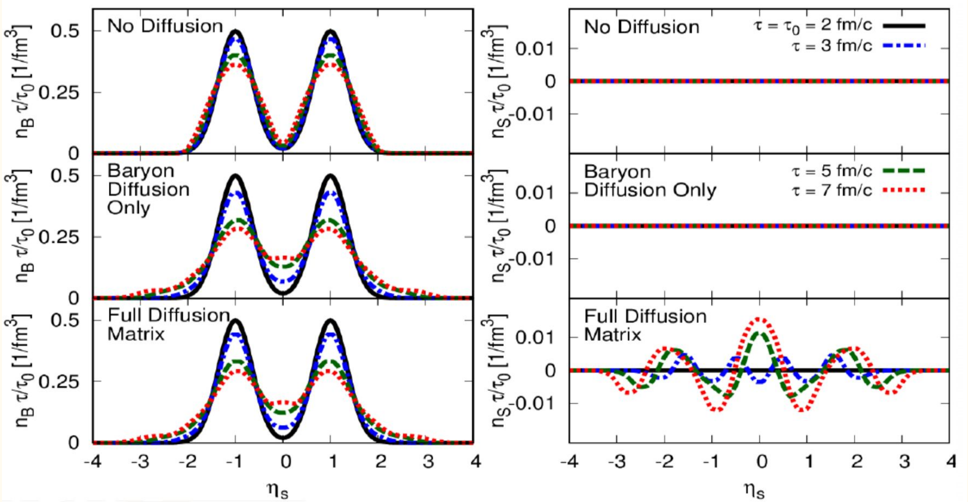
Extra term unique to SPH

Relativistic Coupled Diffusion

Generalization: $for\ q, l \in \{B, S, Q\}$

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)

$$\tau_q \dot{n}_q^\mu + n_q^\mu = \kappa_q \nabla^\mu \alpha_q \Rightarrow \sum_l \tau_{ql} \dot{n}_l^\mu + n_q^\mu = \sum_l \kappa_{ql} \nabla^\mu \alpha_l$$



This needs to be generalized to Israel-Stewart in SPH for our purposes

Full Israel-Stewart With BSQ Diffusion in SPH

$$\tau_\Pi \dot{\Pi} + \Pi = -(\zeta + \tau_\Pi \Pi)\theta - \frac{\tau_\Pi}{2\beta_\Pi}\dot{\beta}_\Pi \Pi - \frac{\zeta}{\beta}\sum_{q,q'}\left(\gamma_0^{qq'} D_\mu n_q^\mu + \frac{1}{2} n_q^\mu (\nabla_\mu \gamma_0^{qq'} - \gamma_0^{qq'}\frac{1}{\sigma}\nabla_\mu \sigma)\right)$$

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = (2\eta\sigma^{\mu\nu} + \tau_\pi \pi^{\mu\nu})\theta - \frac{\tau_\pi}{2\beta_\pi}\dot{\beta}_\pi \pi^{\mu\nu} - \frac{2\eta}{\beta}\sum_{q,q'}\left(\gamma_1^{qq'}\nabla^{\langle\mu} n_q^{\nu\rangle} + \frac{1}{2}(n_q^{\langle\mu}\nabla^{\nu\rangle}\gamma_1^{qq'} - \gamma_1^{qq'} n_q^{\langle\mu}\frac{1}{\sigma}\nabla^{\nu\rangle}\sigma)\right)$$

$$\tau_{qq'}\dot{n}_{q'}^\mu + n_q^\mu = -(\kappa_{qq'}\nabla^\mu \alpha_{q'} + \tau_{qq'} n_{q'}^\mu)\theta - \frac{\tau_{qq'}}{2\beta_{qq'}}\dot{\beta}_{qq'} n_{qq'}^\mu - \frac{\kappa_{qq'}}{\beta}\left(\gamma_0^{qq'}\nabla^\mu \Pi - \frac{\Pi}{2}(\nabla^\mu \gamma_0^{qq'} + 3\gamma_0^{qq'}\frac{1}{\sigma}\nabla^\mu \sigma)\right) - \frac{\kappa_{qq'}}{\beta}\left(\gamma_1^{qq'}\nabla_\nu \pi^{\mu\nu} + \frac{\pi^{\mu\nu}}{2}(\nabla_\nu \gamma_1^{qq'} - \gamma_1^{qq'}\frac{1}{\sigma}\nabla_\nu \sigma)\right)$$

Conclusions and Future Work

- There is still **much** theoretical work and modeling to be done for the upcoming BES 2
  - Hydro Formulation comparison is one area

- These results show that DNMR may offer more robust solutions when the system exhibits criticality

- Studying charge diffusion at LHC energies offers a means of controlled study before moving on to (3+1) BSQ Hydro

Backup Slides

$$\tau_\pi \dot{\pi}_\eta^\eta + \pi_\eta^\eta = \frac{4\eta}{3\tau} - \frac{\eta T \pi_\eta^\eta}{2}\left(\frac{\beta_\pi}{\tau} + \dot{\beta}_\pi\right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{\zeta T \Pi}{2}\left(\frac{\beta_\Pi}{\tau} + \dot{\beta}_\Pi\right)$$

$$\tau_\pi = \frac{5\ \eta}{\epsilon + p}$$

$$\tau_\Pi = \frac{\zeta}{15(\epsilon + p)\left(\frac{1}{3} - c_s^2\right)^2}$$

$$\lambda_{\pi\Pi} = \frac{6}{5}\tau_\pi$$

$$\delta_{\pi\pi} = \frac{4}{3}\tau_\pi$$

$$\tau_{\pi\pi} = \frac{10}{7}\tau_\pi$$

$$\lambda_{\Pi\pi} = \tau_\Pi \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\tau_\Pi$$

$$\delta_{\Pi\Pi} = \frac{2}{3}\tau_\Pi$$

$$\beta_\pi = \frac{\tau_\pi}{2\eta T}$$

$$\beta_\Pi = \frac{\tau_\Pi}{\zeta T}.$$

$$\dot{\epsilon} = -\frac{1}{\tau}\left[\epsilon + p + \Pi - \pi_\eta^\eta\right]$$

$$\tau_\pi \dot{\pi}_\eta^\eta + \pi_\eta^\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \pi_\eta^\eta\left(\delta_{\pi\pi} + \tau_{\pi\pi}\right) + \lambda_{\pi\Pi}\Pi\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left(\zeta + \delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi_\eta^\eta\right)$$

$$\dot{\rho} = -\frac{\rho}{\tau}$$