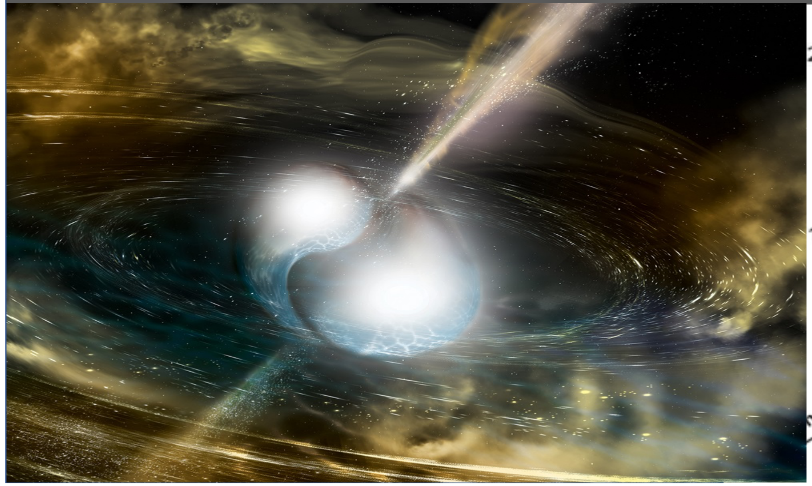
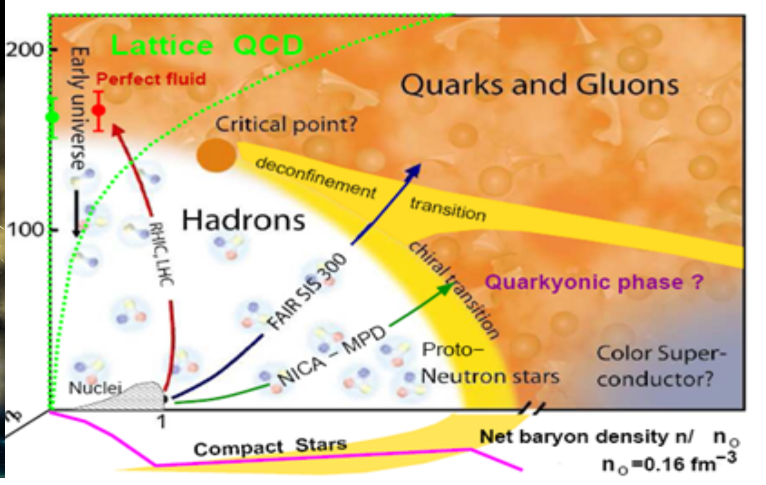
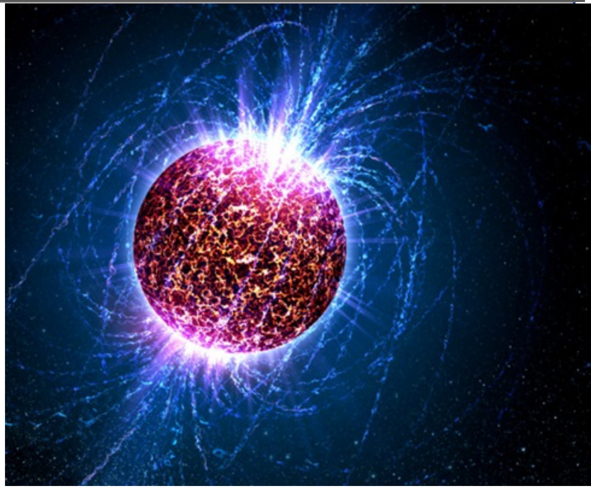
Magnetic Dual Chiral Density Wave Phase of Dense Quark Matter: Properties and Relevance for Neutron Stars







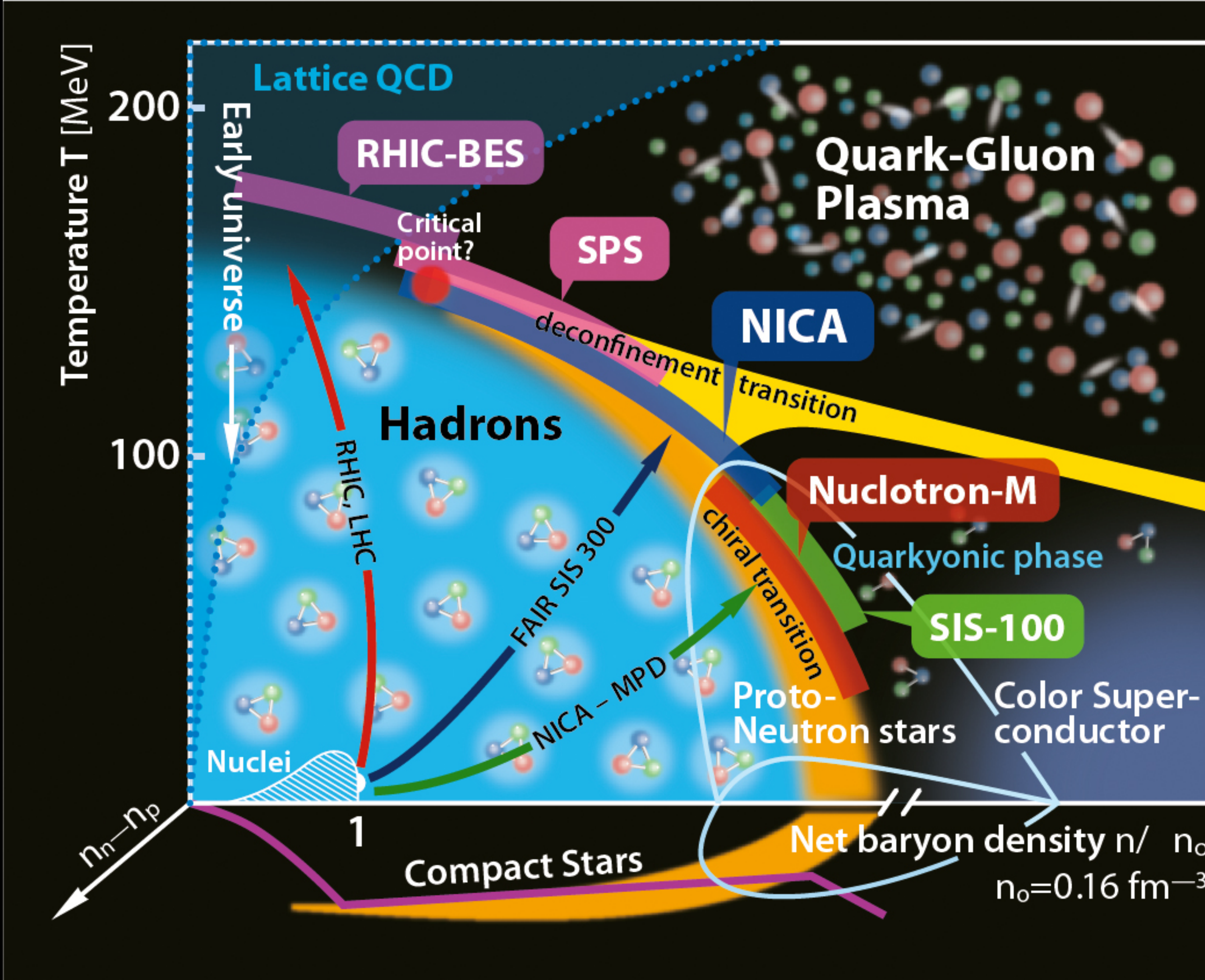
Vivian de la Incera

Sharif University of Technology Webinar
August 4, 2020

Outline

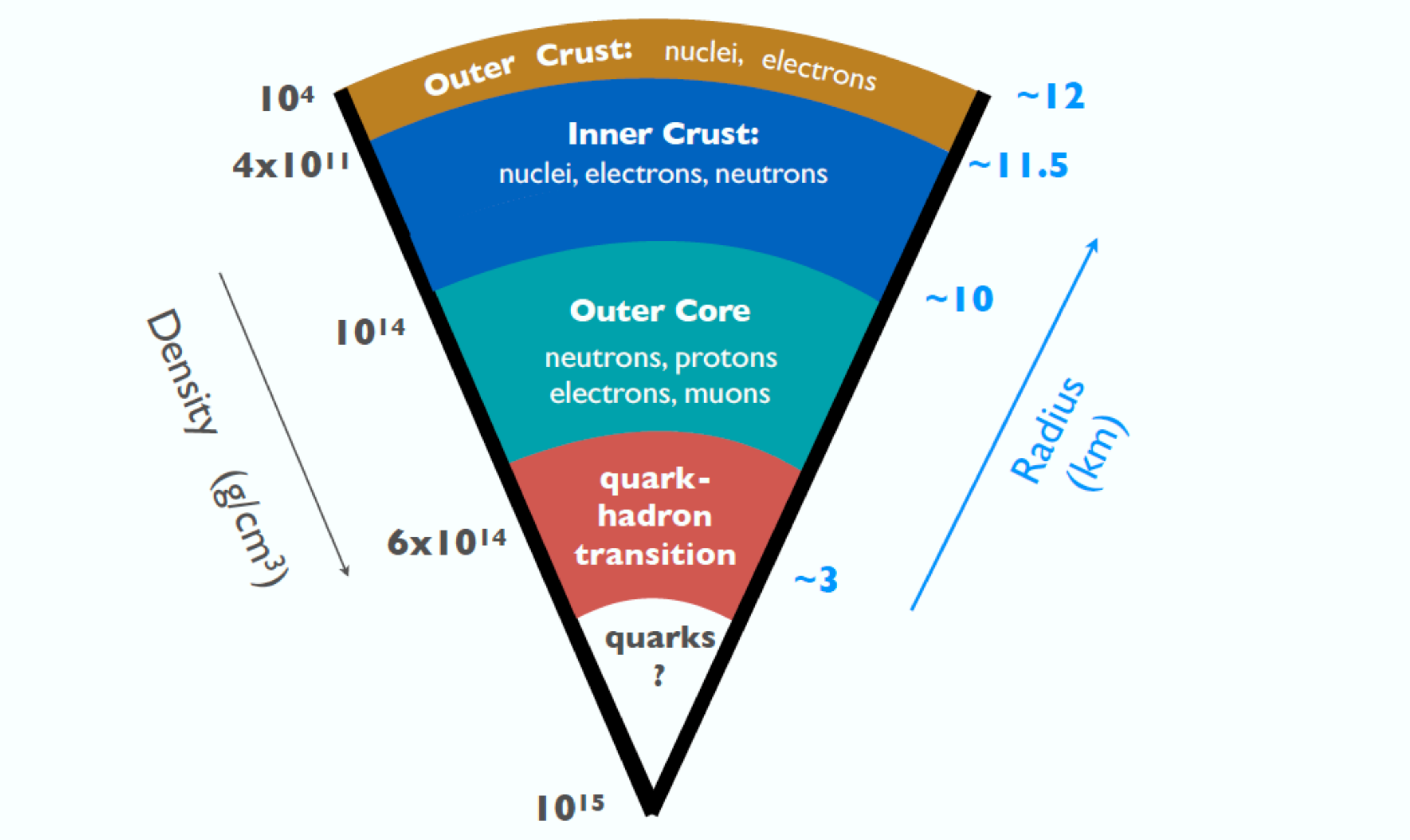
1. Why Quark Matter?
2. Why Inhomogeneous?
3. Basics of the MDCDW Phase
4. MDCDW: a Prospect for NS Core Phases

Matter under Extreme Conditions

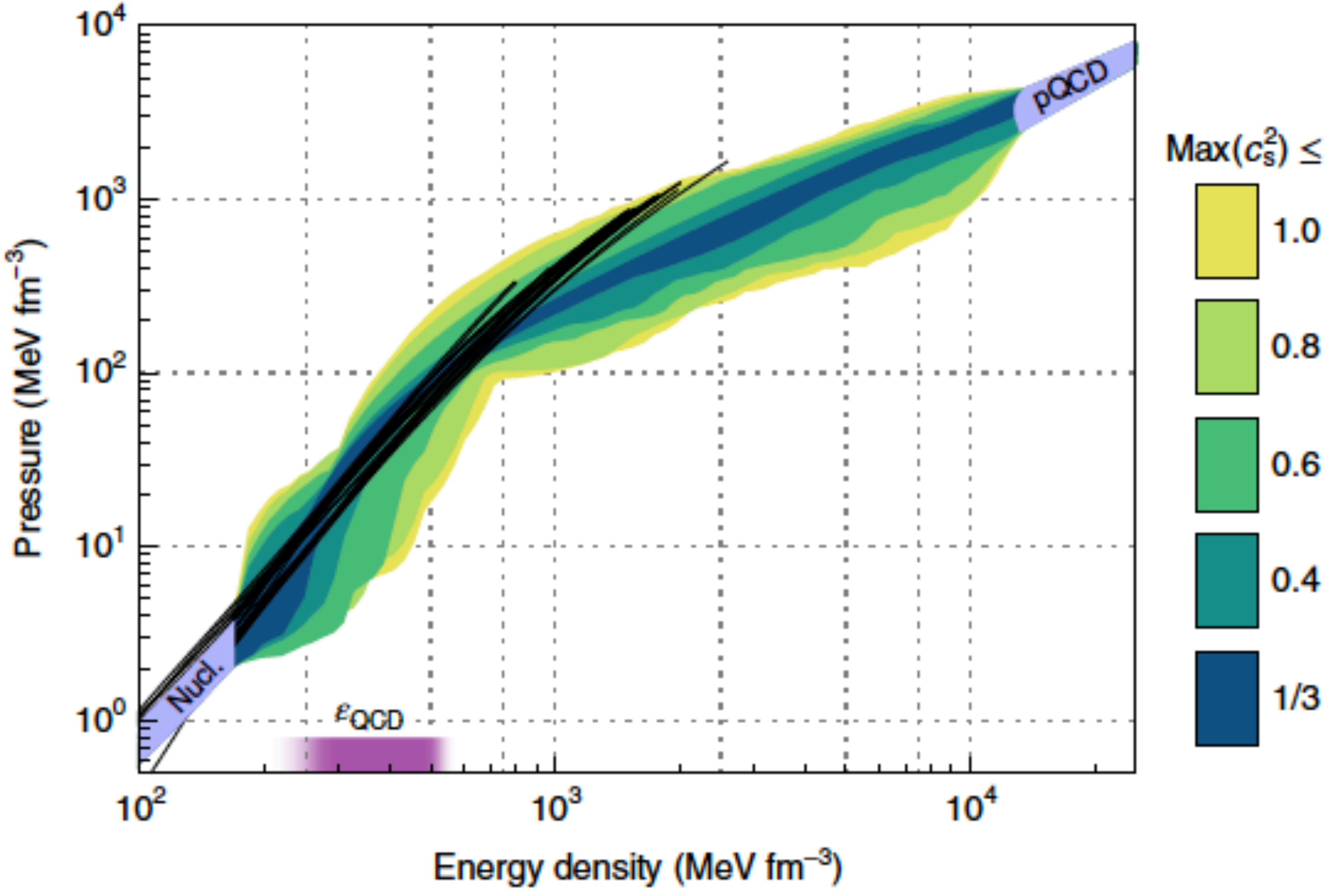


1. Why a Quark Matter Phase?

Composition of Matter in NS



Range of Allowed NS EoS



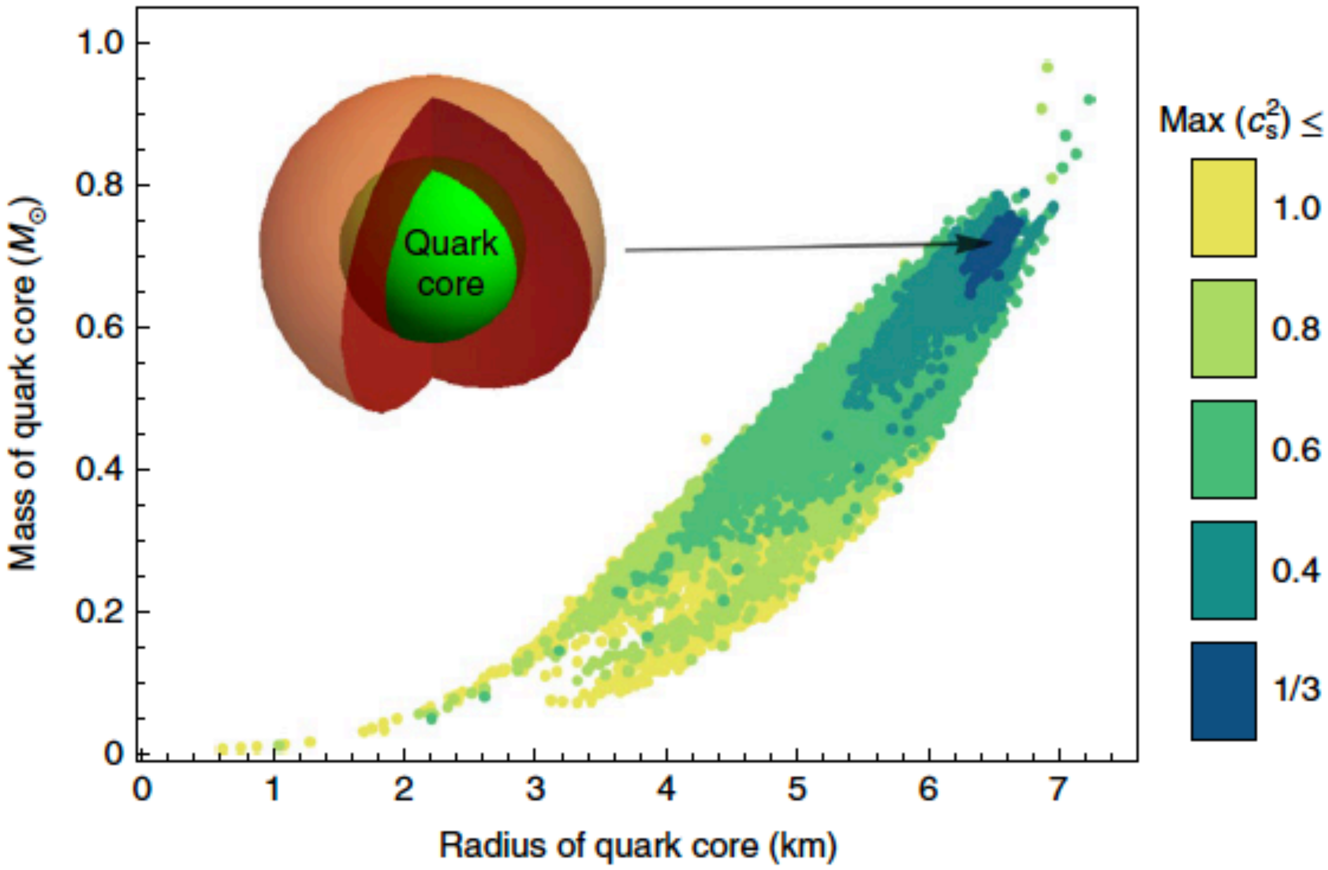
Obtained using multiple interpolation methods and two astrophysical constraints:

- EoS supports $\sim 1.97\, M_{\odot}$
- Tidal deformability

$70 < \Lambda(1.4\ M_{\odot}) < 580$

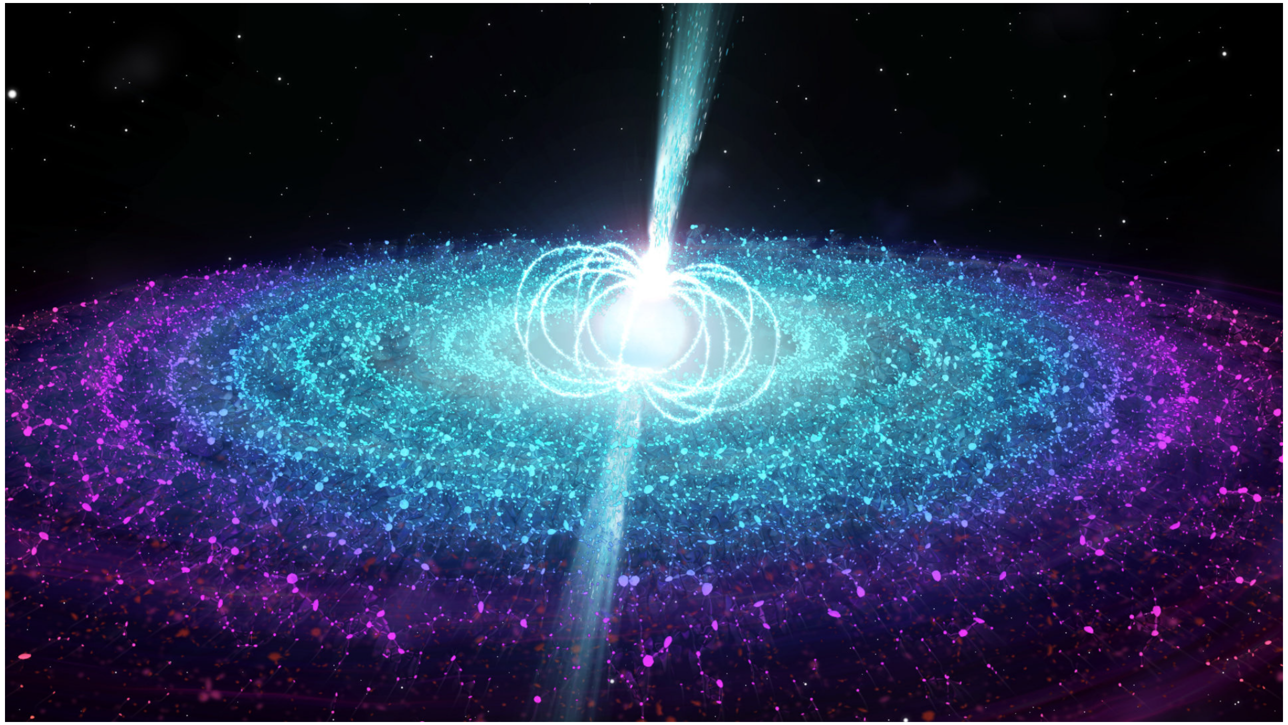
Annala et al., Nature Physics 2020

Quark Matter is Rather Standard



Annala et al., Nature Physics 2020

B is quite Ubiquitous



Pulsar's surface:

$B \sim 10^{12}$–10^{14}G

Magnetars:

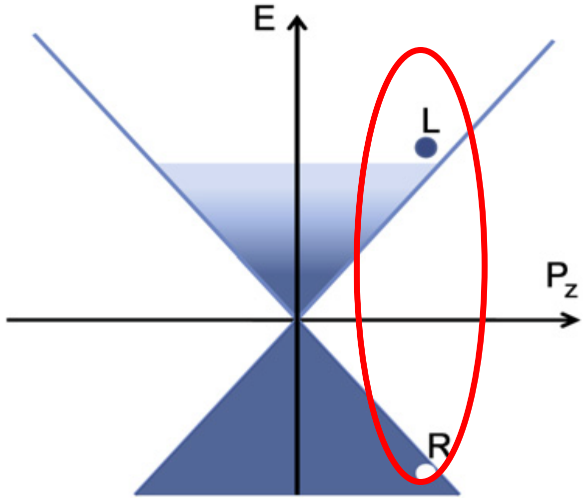
Surface: $B \sim 10^{15}$–10^{16}G

Core: $10^{16}\text{G} < B \leq 10^{18}\text{G}$

2. Why Inhomogeneous?

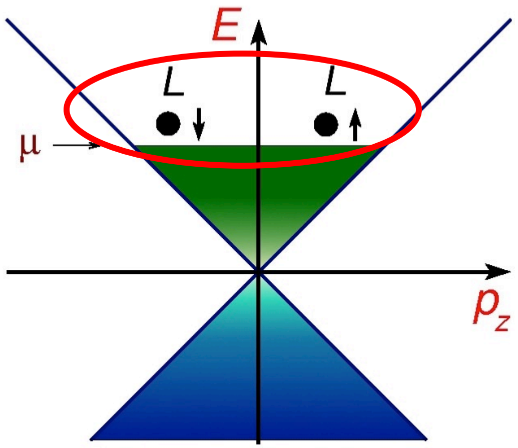
Approaching Intermediate Densities From Both Sides

Chiral Condensate



It pairs particle and antiparticle with opposite spin and momentum (homogeneous condensate)

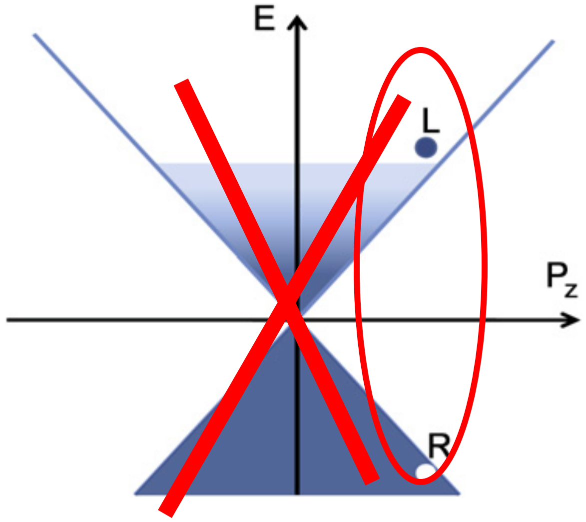
Cooper Pairing



Main channel pairs (all) quarks of different flavors and colors with opposite spins and momenta. Favored at asymptotically high densities.

Approaching Intermediate Densities From Both Sides

Chiral Condensate

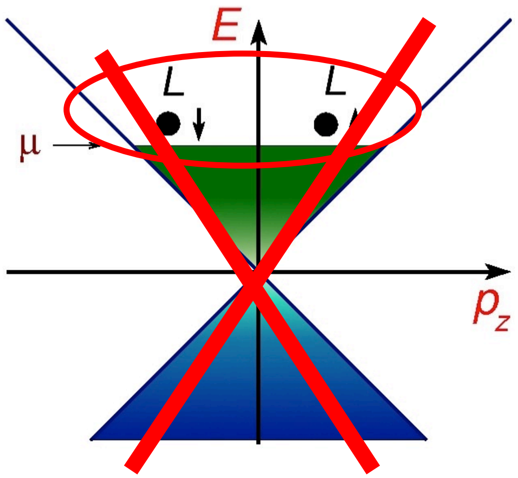


It pairs particle and antiparticle with opposite spin and momentum (homogeneous condensate)

Not favored with increasing density

A Way out: **Spatially Modulated Chiral Condensates**

Cooper Pairing

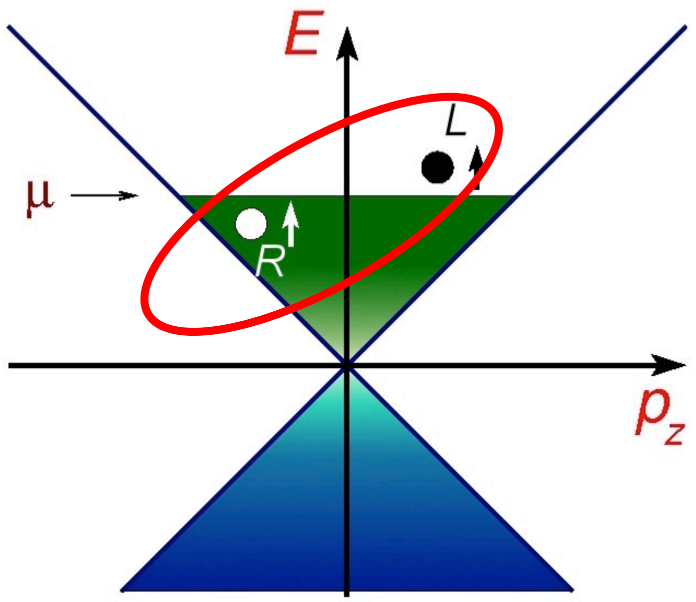


Main channel pairs quarks of different flavors with opposite spins and momenta. Favored at very high densities.

Suffers from Fermi surface mismatch with decreasing densities leading to chromomagnetic instabilities.

A Way out: **Spatially Modulated Quark-Quark Condensates**

Density Wave Pairing



It pairs particle and hole with opposite spin and parallel momenta (nonzero net momentum)

No Fermi surface mismatch

Favored over homogeneous chiral condensate

Favored over CS at large Nc

3. Basics of the MDCDW Phase

Magnetic Dual Chiral Density Wave Model

2-flavor NJL model + QED at finite baryon density and with magnetic field B|| z

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu) + \gamma_0\mu]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\boldsymbol{\tau}\gamma_5\psi)^2].$$

It favors the formation of an inhomogeneous chiral condensate

$$\langle\bar{\psi}\psi\rangle = \Delta\cos q_\mu x^\mu, \qquad \langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = \Delta\sin q_\mu x^\mu \qquad q^\mu = (0,0,0,q)$$

Mean-field Lagrangian

$$\mathcal{L}_{MF} = \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu) + \gamma_0\mu]\psi - m\bar{\psi}e^{i\tau_3\gamma_5 q_\mu x^\mu}\psi - \frac{m^2}{4G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

Magnetic Dual Chiral Density Wave Model

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Complex mass term

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

Chiral Transformation and Asymmetric Spectrum

Performing the chiral transformation

$$\psi \to U_A \psi = e^{-i\tau_3\gamma_5 \frac{qz}{2}} \psi \qquad \bar{\psi} \to \bar{\psi}\bar{U}_A = \bar{\psi} e^{-i\tau_3\gamma_5 \frac{qz}{2}}$$

The MF Lagrangian acquires a mass term plus a $\gamma_3\gamma_5$ term in the derivative

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu 0} + iQA_\mu - i\tau_3\gamma_5\delta_{\mu 3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

The corresponding fermion spectrum is

$$E_k^{LLL} = \epsilon\sqrt{\Delta^2 + k_3^2} + q/2, \quad \epsilon = \pm$$

LLL mode is Asymmetric!

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1, 2, 3, \ldots$$

Nontrivial Topology of the MDCDW Phase

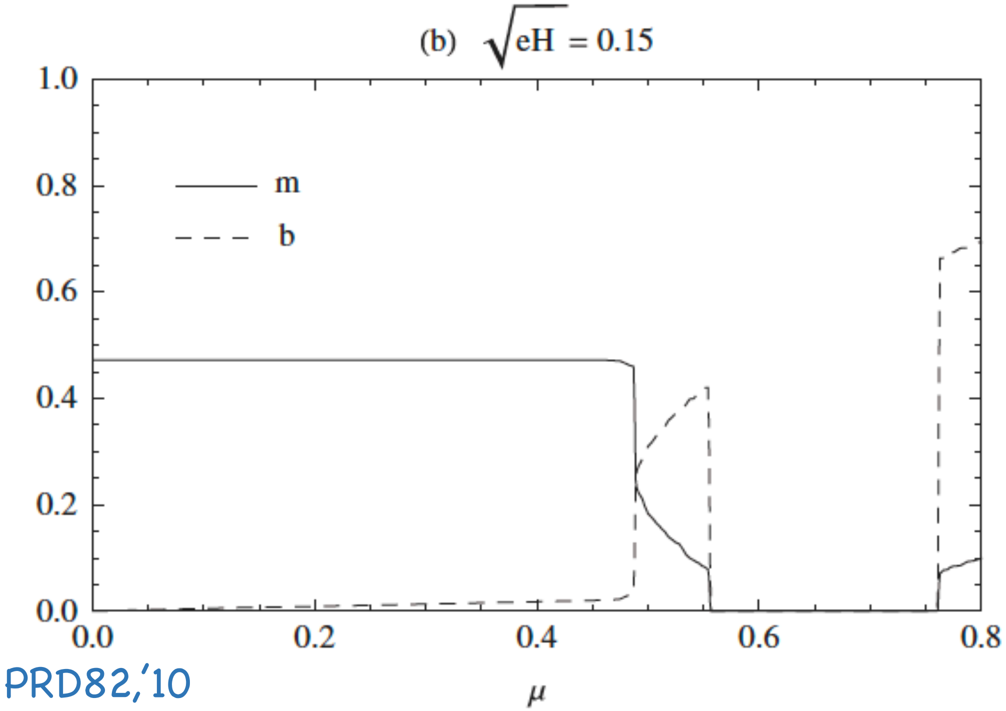
Topology emerges due to the LLL spectral asymmetry

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_T(B,\mu,T) + \frac{m^2}{4G}.$$

$$\Omega^f_{anom} = -\frac{N_c|e_f B|}{(2\pi)^2} q\mu$$

$$\rho^A_B = 3\frac{|e|}{4\pi^2} qB$$

Anomalous baryon number density



The anomaly makes the MDCDW solution energetically favored over the homogeneous condensate

Frolov, et al PRD82,'10

Axion Term

Ferrer & VI, PLB' 2017; NPB' 2018

Key observation: the fermion measure is not invariant under U_A

$$D\bar{\psi}D\psi \rightarrow (\det U_A)^{-2} D\bar{\psi}D\psi \qquad (\det U_A)_R^{-2} = e^{i\int d^4x \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}}$$

$\theta = \frac{qz}{2}$

$\kappa = \frac{e^2}{2\pi^2}$

The effective MF Lagrangian acquires an axion term:

$$\mathcal{L}_{eff} = \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu} - i\tau_3\gamma_5\partial_{\mu}\theta) + \gamma_0\mu - m]\psi - \frac{m^2}{4G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu},$$

Integrating out the fermions, we find the electromagnetic effective action in the MDCDW model

$$\Gamma(A) = V\Omega + \int d^4x \left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}\right] - \int d^4x A^{\mu}(x)J_{\mu}(x) + \cdots,$$

QED in MDCDW is Axion QED

$$\nabla \cdot E = J_0 + \frac{e^2}{4\pi^2} qB$$

Anomalous charge

$$\nabla \times B - \frac{\partial E}{\partial t} = J_V + \frac{e^2}{4\pi^2} q \times E$$

$$\nabla \cdot B = 0 \quad \nabla \times E + \frac{\partial B}{\partial t} = J_V$$

Dissipationless Hall current
⊥ to both B and E

Anomalous Hall conductivity

$$\sigma_{xy}^{anom} = e^2 q / 4\pi^2$$

Ferrer & VI, Phys.Lett. B769 (2017) 208; Nucl.Phys. B931 (2018) 192

4. Can MDCDW be a viable Prospect for NS Core Phases?

Test #1: Stability against Fluctuations

Low-energy Theory

Order Parameter

$$M(x) = \sigma(x) + i\pi(x)$$

$$\sigma = -2G\bar{\psi}\psi \qquad \pi = -2G\bar{\psi}i\gamma^5\tau_3\psi,$$

The low-energy theory is described by a generalized GL expansion of the thermodynamic potential in powers of the order parameter and its derivatives. The resulting expansion is invariant under the symmetries of the original theory in a B

$U_V(1) \times U_A(1) \times SO(2) \times R^3$

GL Expansion

Ferrer & VI, PRD'2020

Low Energy GL Expansion of the MDCDW Free Energy

$$
\begin{aligned}
\mathcal{F} &= a_{2,0}|M|^2 - i\frac{b_{3,1}}{2}[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] + a_{4,0}|M|^4 + a_{4,2}^{(0)}|\nabla M|^2 \\
&+ a_{4,2}^{(1)}(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) - i\frac{b_{5,1}}{2}|M|^2[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] \\
&+ \frac{ib_{5,3}}{2}[(\nabla^2 M^*)\hat{B}\cdot\nabla M - \hat{B}\cdot\nabla M^*(\nabla^2 M)] + a_{6,0}|M|^6 + a_{6,2}^{(0)}|M|^2|\nabla M|^2 \\
&+ a_{6,2}^{(1)}|M|^2(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) + a_{6,4}|\nabla^2 M|^2 + \cdots.
\end{aligned}
$$

MDCDW ansatz

$$M(z) = me^{iqz}$$

$$\rightarrow$$

$$
\begin{aligned}
\mathcal{F} &= a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4 \\
&+ b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2,
\end{aligned}
$$

The b coefficients are a consequence of the asymmetry of the LLL spectrum

The $a_{x,y}^{(1)}$ coefficients are a consequence of having an external vector

GL Expansion

Ferrer & VI, PRD'2020

Low Energy GL Expansion of the MDCDW Free Energy

$$\mathcal{F} = a_{2,0}|M|^2 - i\frac{b_{3,1}}{2}[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] + a_{4,0}|M|^4 + a_{4,2}^{(0)}|\nabla M|^2$$
$$+ a_{4,2}^{(1)}(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) - i\frac{b_{5,1}}{2}|M|^2[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M]$$
$$+ \frac{ib_{5,3}}{2}[(\nabla^2 M^*)\hat{B}\cdot\nabla M - \hat{B}\cdot\nabla M^*(\nabla^2 M)] + a_{6,0}|M|^6 + a_{6,2}^{(0)}|M|^2|\nabla M|^2$$
$$+ a_{6,2}^{(1)}|M|^2(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) + a_{6,4}|\nabla^2 M|^2 + \cdots.$$

MDCDW ansatz

$$M(z) = me^{iqz}$$

$$\mathcal{F} = a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4$$
$$+ b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2,$$

The b coefficients are a consequence of the asymmetry of the LLL spectrum

The $a_{x,y}^{(1)}$ coefficients are a consequence of having an external vector

Spontaneous Breaking of Chiral and Translational Symmetries

$\bar{M}(z) = m e^{iqz}$ with m and q solutions of the stationary equations:

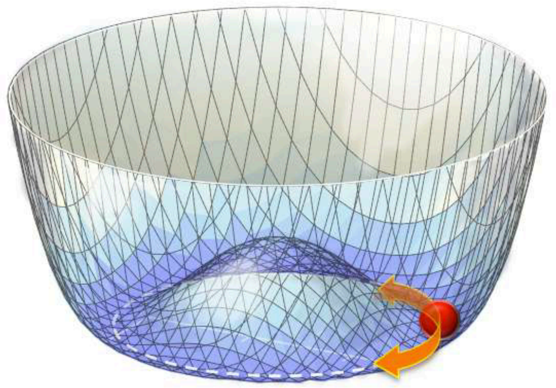
$$\partial\mathcal{F}/\partial m = 2m\{a_{2,0} + 2a_{4,0}m^2 + 3a_{6,0}m^4 + q^2[a_{4,2} + 2a_{6,2}m^2 + a_{6,4}q^2] + q[b_{3,1} + 2b_{5,1}m^2 + b_{5,3}q^2]\} = 0,$$

$$\partial\mathcal{F}/\partial q = m^2\{2q[a_{4,2} + a_{6,2}m^2 + 2a_{6,4}q^2] + b_{3,1} + b_{5,1}m^2 + 3b_{5,3}q^2\} = 0.$$

$$a_{4,2} = a_{4,2}^{(0)} + a_{4,2}^{(1)}, \quad a_{6,2} = a_{6,2}^{(0)} + a_{6,2}^{(1)}$$

Symmetry is reduced to $U_V(1) \times SO(2) \times R^2$

Fluctuations of the condensate come from two Goldstone Bosons: pions and phonons



Low-Energy Theory of Fluctuations

Chiral and translation transformations are locked

$$M(x) \rightarrow e^{i\tau} M(z+\xi) = e^{i(\tau+q\xi)} M(z)$$

The phonon free energy is then

$$\mathcal{F}[M(x)] = \mathcal{F}_0 + v_z^2(\partial_z\theta)^2 + v_\perp^2(\partial_\perp\theta)^2 + \zeta^2(\partial_z^2\theta + \partial_\perp^2\theta)^2$$

$$v_z^2 = a_{4,2} + m^2 a_{6,2} + 6q^2 a_{6,4} + 3qb_{5,3}, \quad v_\perp^2 = a_{4,2} + m^2 a_{6,2} + 2q^2 a_{6,4} + qb_{5,3} - a_{4,2}^{(1)} - m^2 a_{6,2}^{(1)}.$$

Anisotropic spectrum $\quad E \simeq \sqrt{v_z^2 k_z^2 + v_\perp^2 k_\perp^2}$

Stability against the Fluctuations

Ferrer & VI, PRD'2020

$$\langle M \rangle = m e^{iqz} \langle \cos qu \rangle \qquad \langle \cos qu \rangle = e^{-\langle (qu)^2 \rangle /2}$$

$$\langle q^2 u^2 \rangle = \frac{1}{(2\pi)^2} \int_0^\infty dk_\perp k_\perp \times \int_{-\infty}^{\infty} dk_z \frac{T}{m^2 (v_z^2 k_z^2 + v_\perp^2 k_\perp^2 + \zeta^2 k^4)} \simeq \frac{\pi T}{m\sqrt{v_z^2 v_\perp^2}},$$

Finite! Thanks to B there are no soft transverse modes, hence no Landau-Peierls instability. The MDCDW phase **is stable** against thermal fluctuations.

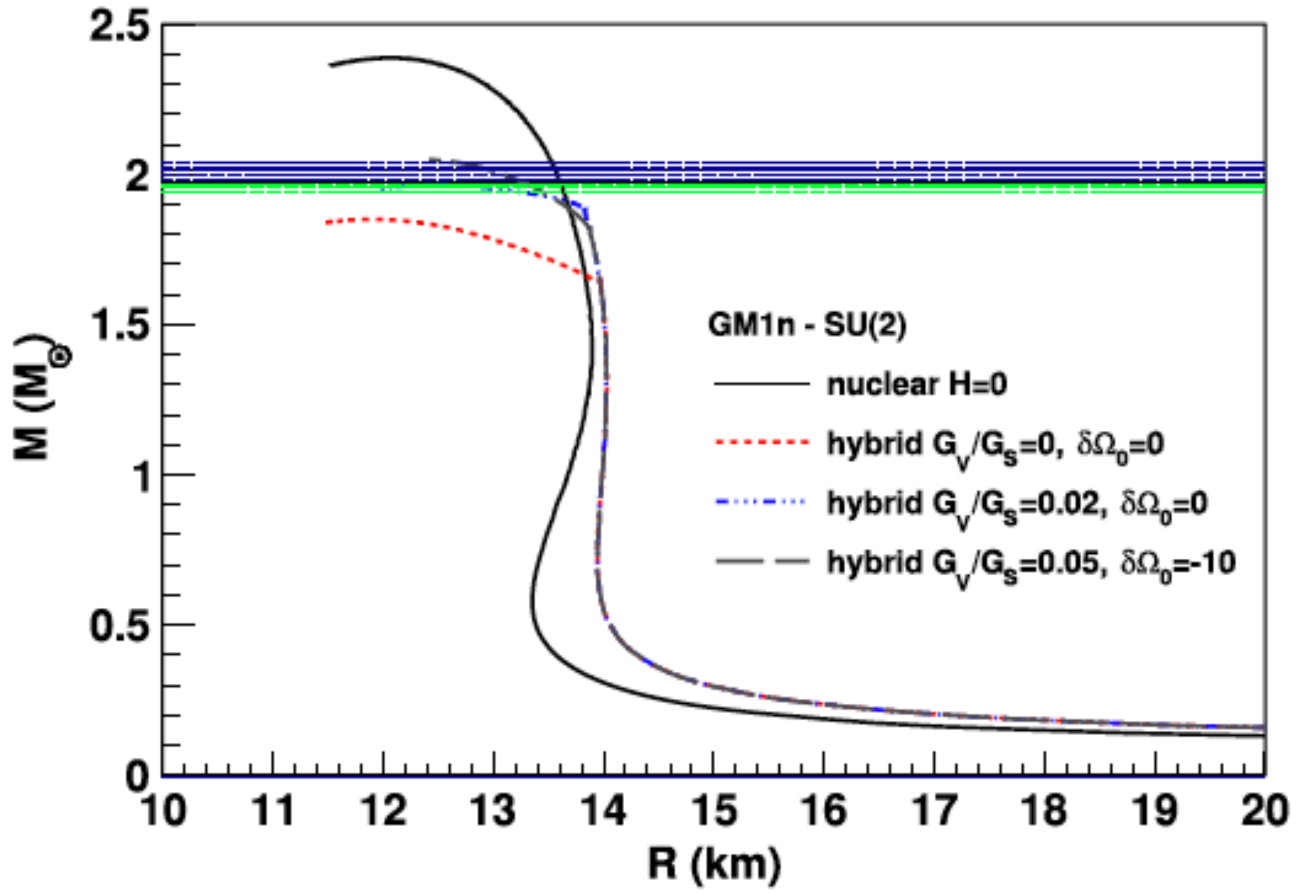
In contrast at B=0

$$\langle u^2 \rangle \simeq \frac{T}{4\pi v_z \zeta} \ln\left(\frac{v_z l_\perp}{\zeta}\right)$$

Infrared divergent. Any finite T no matter how small destroy the long-range order

Test #2: 2 $M_\odot$

Electrically Neutral MDCDW Phase



Carignano, Ferrer, VI, Paulucci, PRD 2015

Test #3: Thermal Properties

Work in Progress

Lower limit of NS core heat capacity established from transiently-accreting NS

$$\frac{C}{\bar{T}_8} > 3.1 \times 10^{36}\,\mathrm{erg\,K^{-1}} \left(\frac{\bar{T}_7}{7}\right)^{-2} \left(\frac{E}{7.5 \times 10^{43}\,\mathrm{erg}}\right).$$

is violated by CFL

Cummings et. al PRC'2017

But not by MDCDW. Paper coming soon...

MDCDW

Described by Dirac Hamiltonian

$$H_f = -i\gamma^0\gamma^i(\partial_i + ie_f A_i + i\frac{e_f}{|e_f|}\gamma_5\partial_i\theta) + \gamma^0 m$$

Axion term in the electromagnetic action

$$S = -\kappa \int d^4x \epsilon^{\mu\alpha\nu\beta} A_\alpha \partial_\nu A_\beta \partial_\mu \theta$$

Topology is associated to asymmetry of the LLL states in the MDCDW.

$$\sigma_{xy}^{anom} = e^2 q/4\pi^2$$

Anomalous Hall conductivity

Ferrer and VI, '15,'17,'18

Weyl Semimetals

Described by Dirac Hamiltonian

$$H(\mathbf{k}) = \gamma^0\gamma^i(k_i - b_i\gamma^5) + m\gamma^0 + b_0\gamma^5.$$

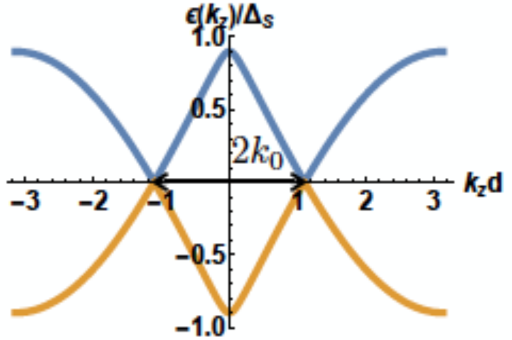
Axion term in the electromagnetic action

$$S = -\frac{e^2}{4\pi^2}\int dt\, d^3r\, b_\mu \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta.$$

Topology is associated to band structure with nodes of opposite chirality separated by 2b in momentum space

$$\sigma_{xy} = \frac{e^2}{h}\frac{2|b|}{2\pi^2}$$

Anomalous Hall conductivity



Burkov, '17

Summary:

- MDCDW is a viable phase at intermediate densities
- So far is compatible with several astrophysical constraints

Outlook:

- Need to Connect to more Measurable NS observables that can confirm/falsify proposed intermediate density candidates: MDCDW, Quarkyonic, CS Phases.
- More studies on consequences of the topological properties