

Vorticity and spin polarization in heavy-ion collisions

Xu-Guang Huang

Fudan University, Shanghai

**September 15th , 2020 @ Webnar given at
Sharif University of Technology, Tehran, Iran**

Motivation of the talk

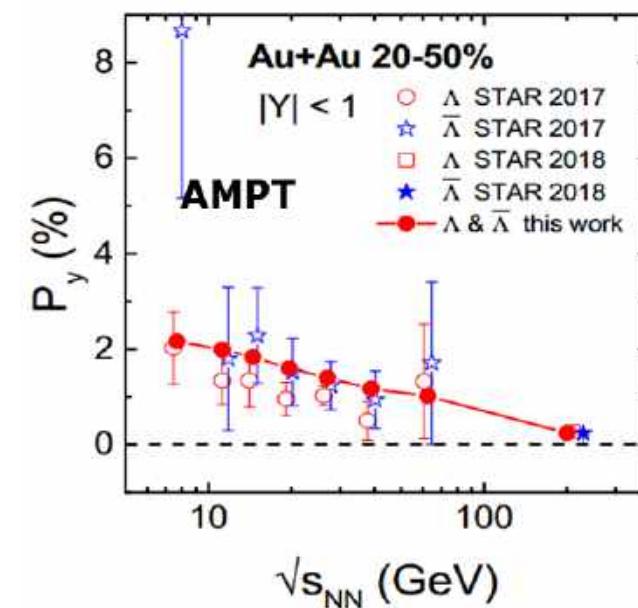
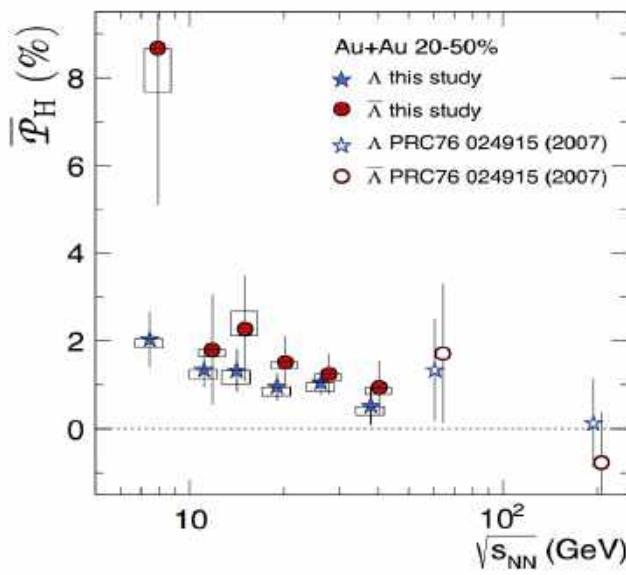
LETTER

Quark-gluon plasma: “The most vortical fluid”

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*



Experiment

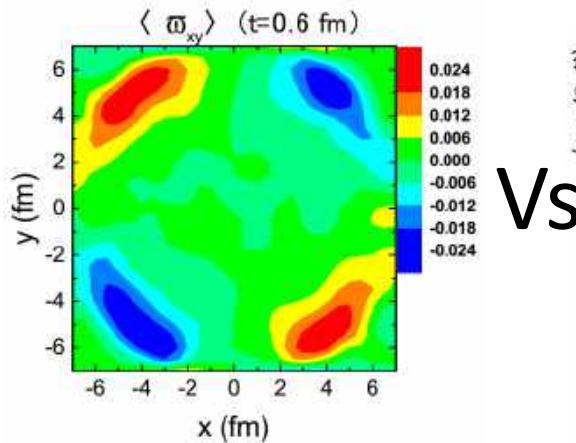
=

Theory

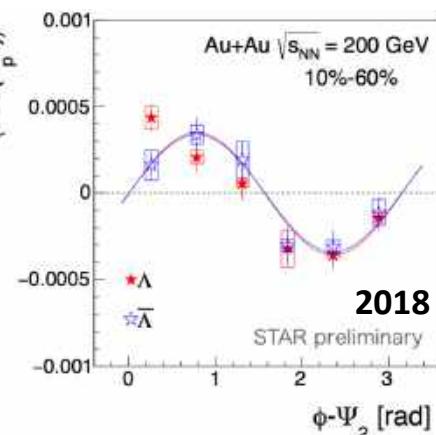
Motivation of the talk

- But: discrepancies exist between theory and experiments

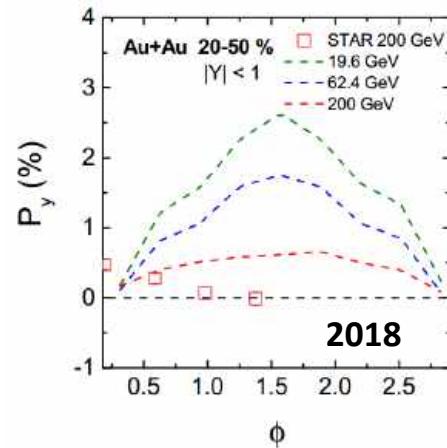
1) longitudinal polarization vs ϕ



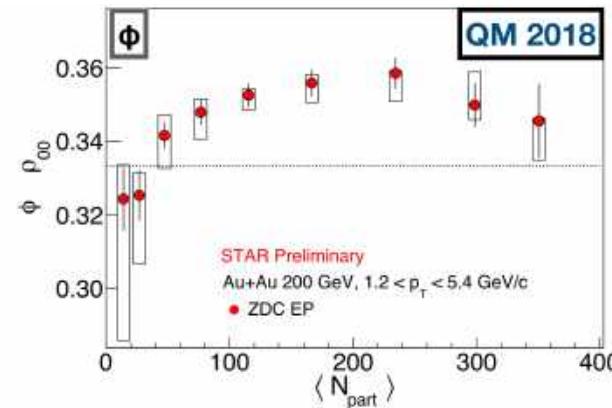
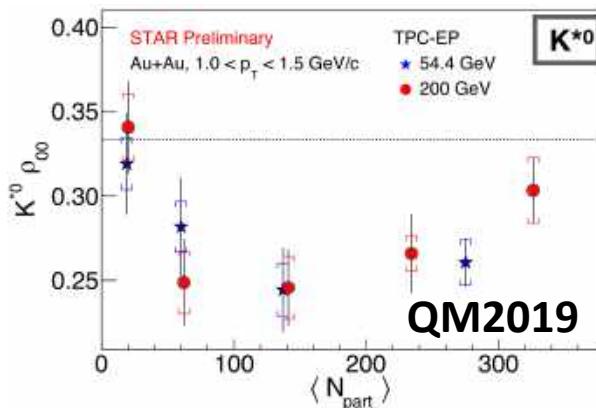
VS



2) Transverse polarization vs ϕ



3) Vector meson spin alignment



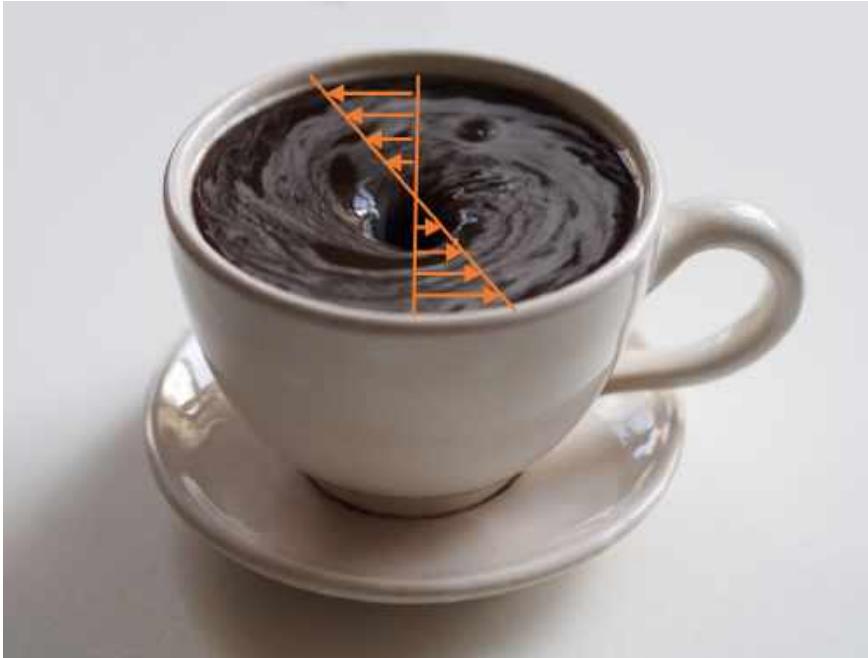
Too big than expected!
Sign is not understood!

Outline

- Vorticity in heavy-ion collisions (HICs)
- From vorticity to spin polarization of hadrons
- Spin hydrodynamics
- Spin alignment and spin dependent hadron yields
- Summary

Vorticity in heavy-ion collisions

What is vorticity?



$$\omega = \frac{1}{2} \nabla \times \mathbf{v}$$

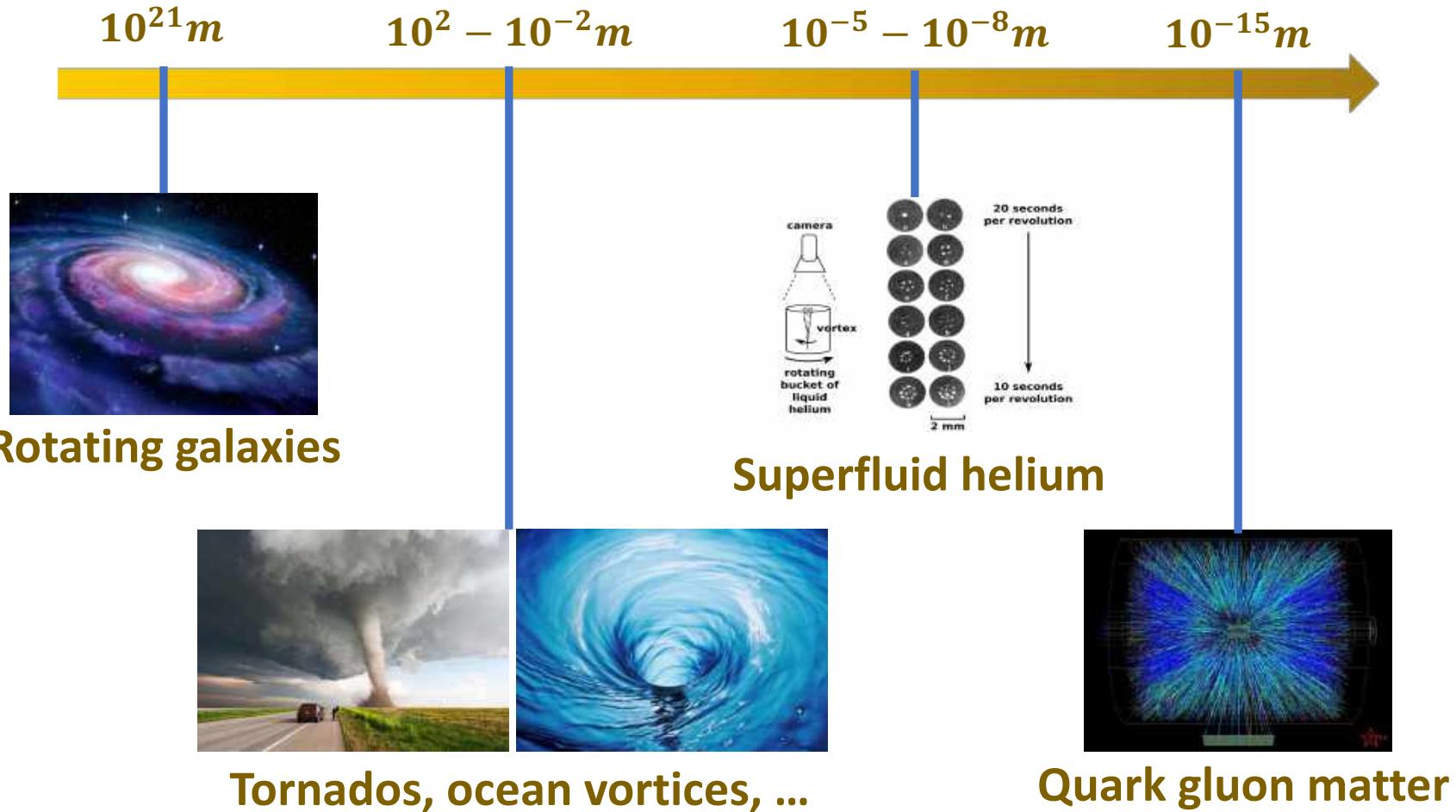
Fluid vorticity

Vortex in a coffee cup

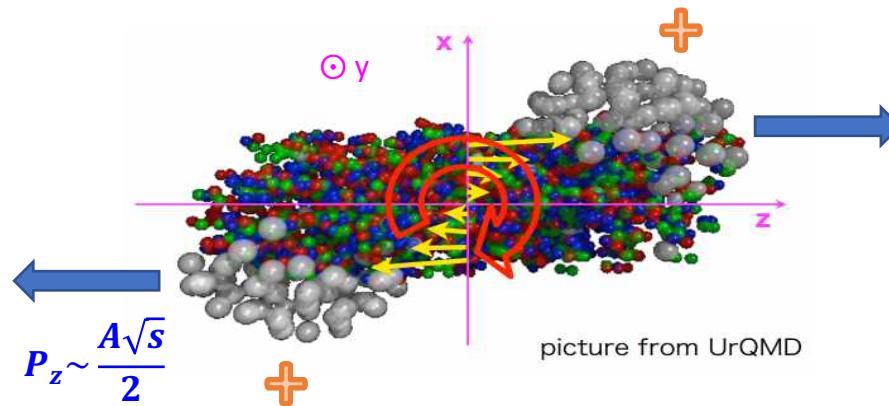
(Angular velocity of fluid cell)

What is vorticity?

- Vortices: common phenomena in fluids across a very broad hierarchy of scales



Why fluid vorticity in HICs?



$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

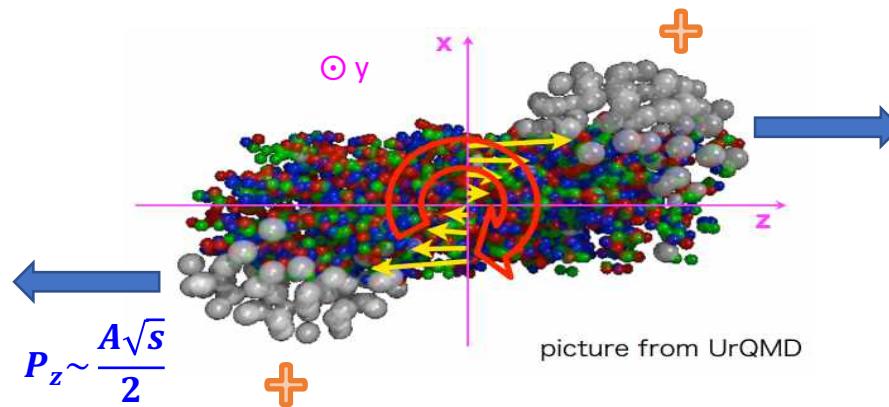
Global angular momentum

$$eB \sim \gamma \alpha_{\text{EM}} \frac{z}{b^2} \sim 10^{18} \text{ G}$$

Strong Magnetic field

(RHIC Au+Au 200 GeV, $b=10$ fm)

Why fluid vorticity in HICs?



picture from UrQMD

$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

Global angular momentum

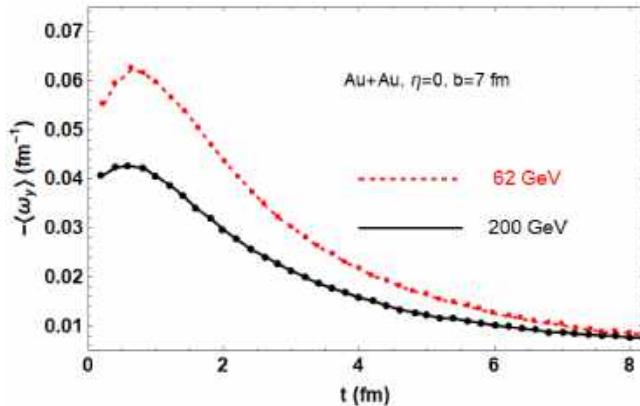
Local vorticity

$$\omega \sim ?$$

(RHIC Au+Au 200 GeV, $b=10$ fm)

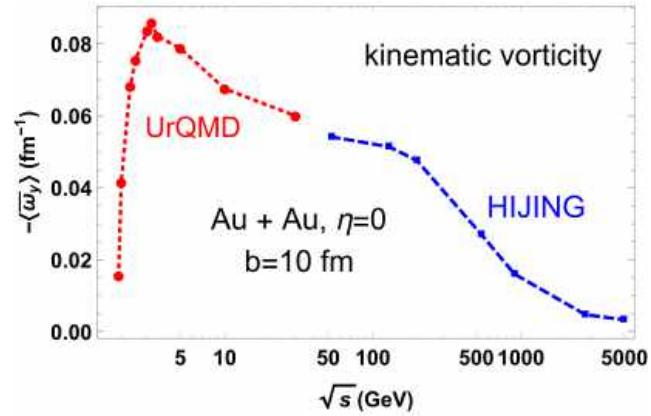
Vorticity by global angular momentum

Time dependence



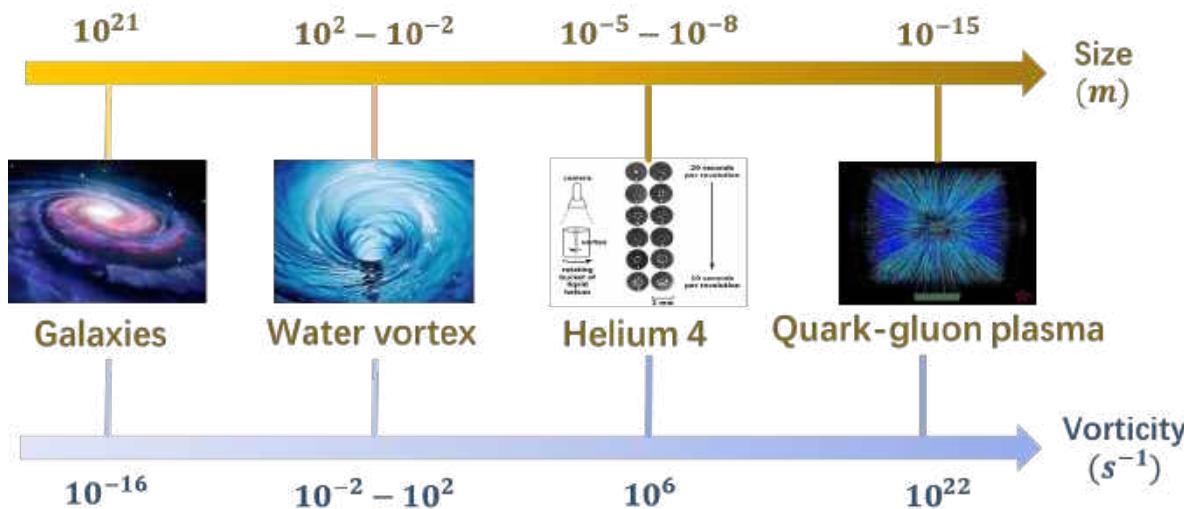
AMPT (Jiang-Lin-Liao 2016)

Energy dependence of initial vorticity



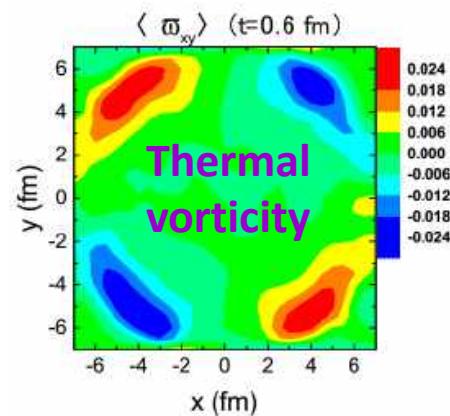
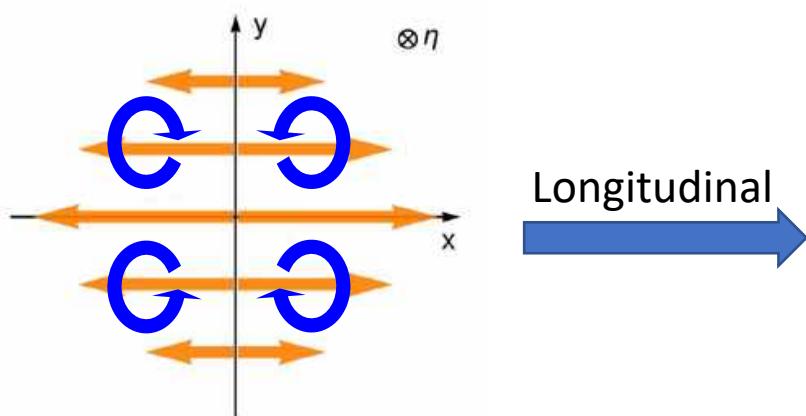
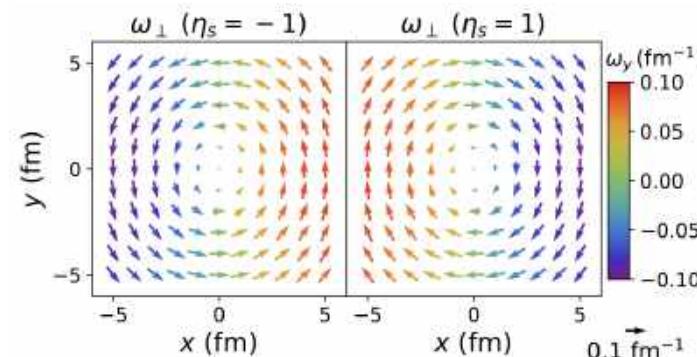
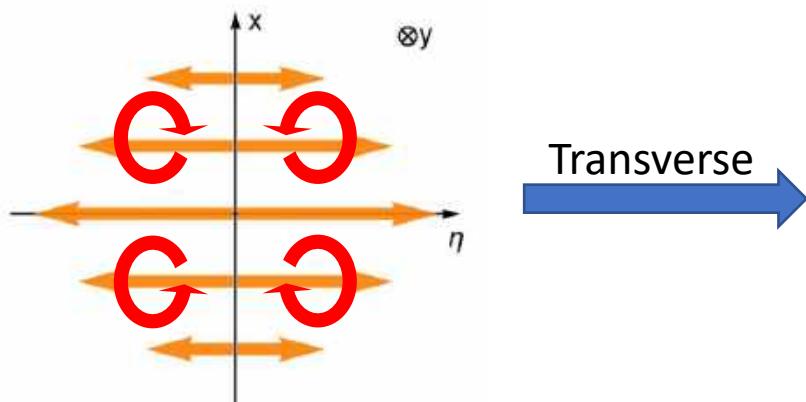
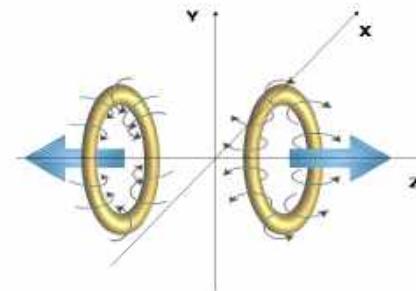
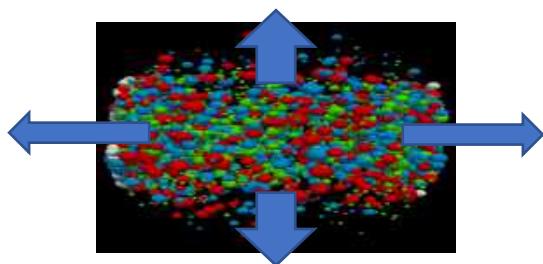
(Deng-XGH 2016; Deng-XGH-Ma-Zhang 2020)

The most vortical fluid: Au+Au@RHIC at $b=10$ fm is $10^{20} - 10^{21} \text{ s}^{-1}$



(See also: Becattini-Karpenko et al 2015,2016; Xie-Csernai et al 2014,2016,2019; Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; Wei-Deng-XGH 2018;)

Vorticity by fireball expansion

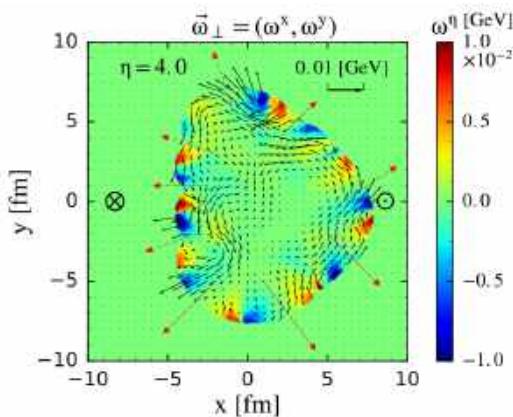
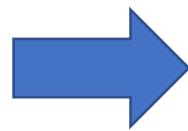
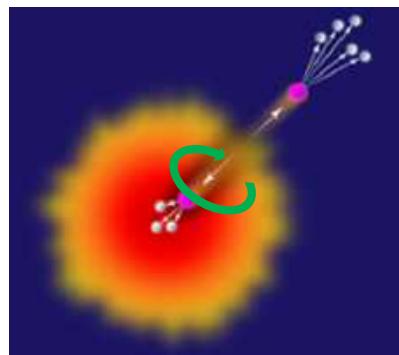


(Wei-Deng-XGH 2018)

(See also: Karpenko-Becattini 2017; Csernai et al 2014; Teryaev-Usubov 2015; Ivanov-Soldatov 2018;)

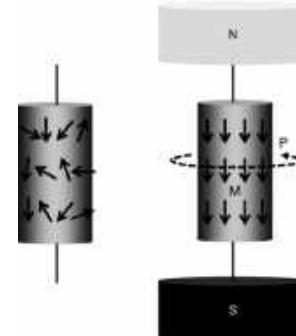
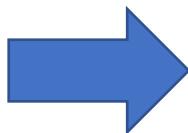
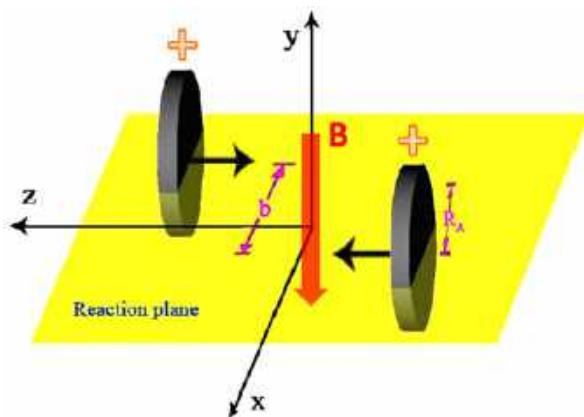
Other sources of vorticity

1) Jet



(Pang-Peterson-Wang-Wang 2016)

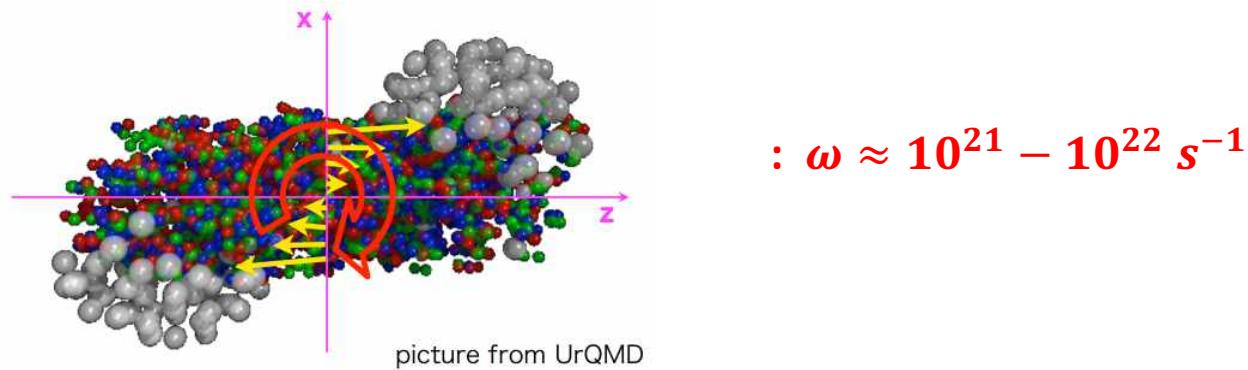
2) Magnetic field



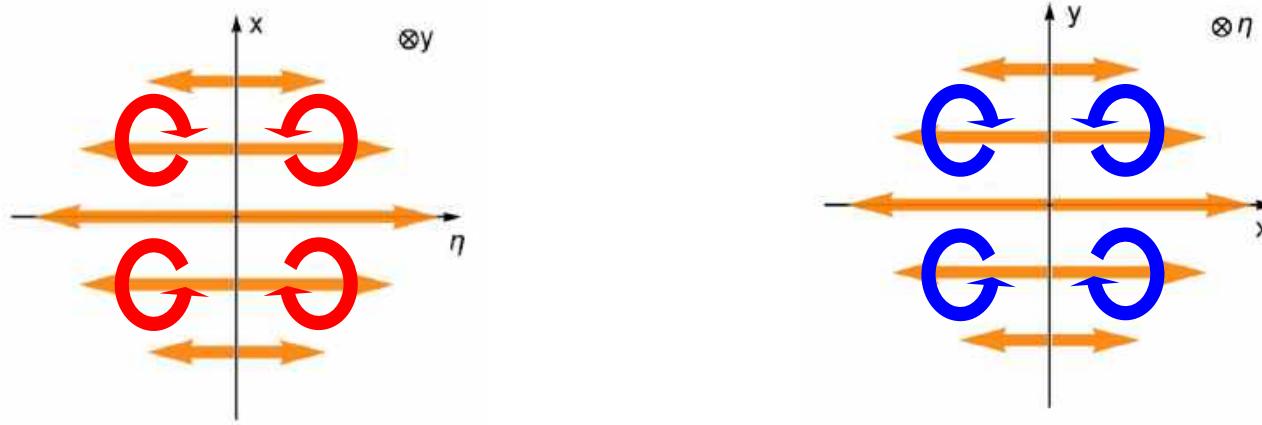
Einstein-de Haas effect

Main message:

1. Global AM induces strong vorticity in HICs



2. Fireball expansion: quadrupoles in both xy and xz planes



How to detect it experimentally? Spin polarization, Chiral vortical effects,

Spin polarization by vorticity

How vorticity polarizes spin?

Early idea: Liang-Wang PRL2005; Voloshin 2004

Vorticity interpretation (at thermal equilibrium)

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S} \quad \xrightarrow{\hspace{1cm}} \quad \frac{dN}{dp} \sim e^{-(H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S})/T}$$
$$\quad \xrightarrow{\hspace{1cm}} \quad P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

More rigorous derivation (Becattini et al 2013; Fang et al 2016; Liu-Mamedya-XGH 2020)

$$P^\mu(p) = \frac{1}{4m} \epsilon^{\mu\nu\rho\sigma} p_\nu \frac{\int d\Sigma_\lambda p^\lambda f'(x, p) \varpi_{\rho\sigma}(x)}{\int d\Sigma_\lambda p^\lambda f(x, p)} + O(\varpi^2)$$

- Valid at **global equilibrium**. $f(x, p)$ is the distribution function (Fermi-Dirac)
- Thermal vorticity $\varpi_{\rho\sigma} = \left(\frac{1}{2}\right) (\partial_\sigma \beta_\rho - \partial_\rho \beta_\sigma)$
- Spin polarization is enslaved to thermal vorticity, not dynamical
- Friendly for numerical simulation (a spin Cooper-Frye type formula)

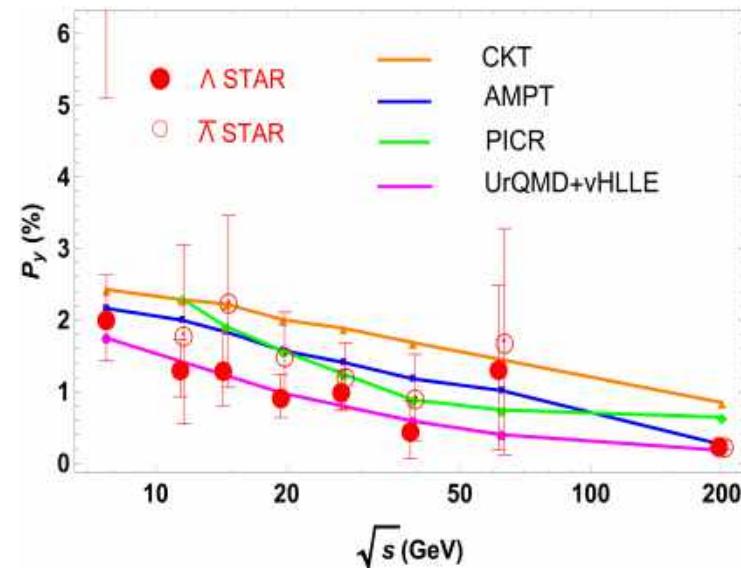
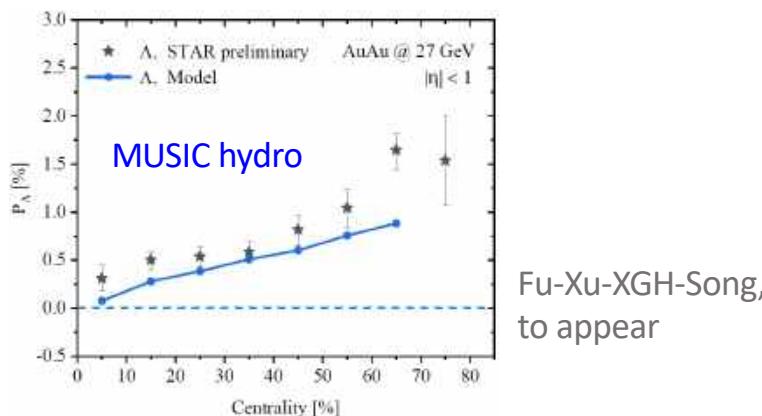
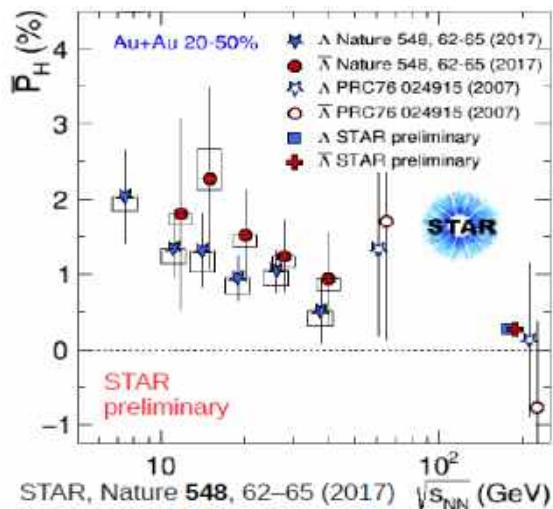
Global Λ spin polarization

The global polarization (i.e., integrated polarization over kinematics):

Experiment

=

Theory



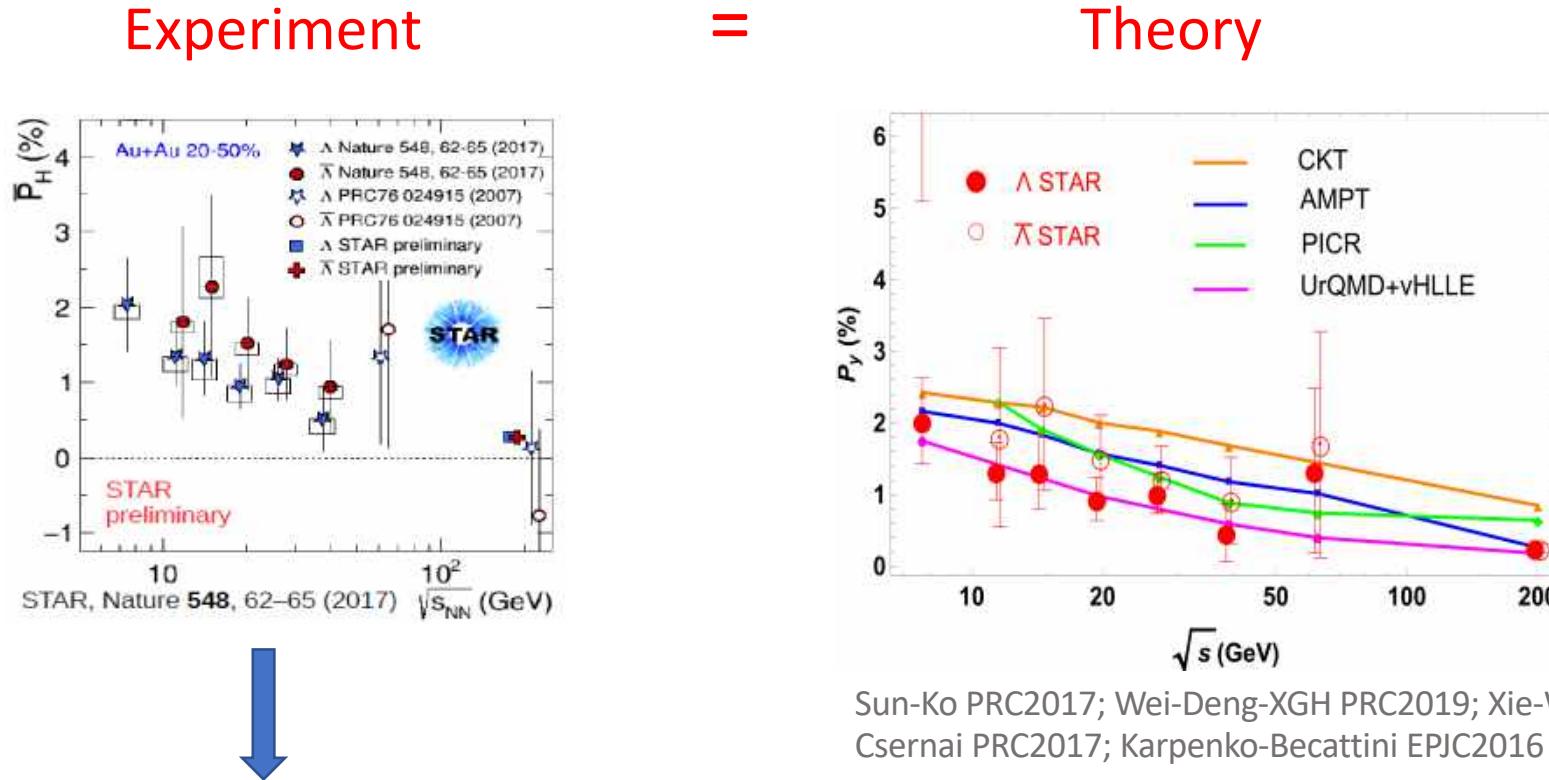
Sun-Ko PRC2017; Wei-Deng-XGH PRC2019; Xie-Wang-Csernai PRC2017; Karpenko-Becattini EPJC2016

(Many similar results in literature)

Vorticity interpretation of global Λ polarization works well!

Global Λ spin polarization

The global polarization (i.e., integrated polarization over kinematics):



Though with big error bar, a difference between $P_y(\Lambda)$ and $P_y(\bar{\Lambda})$ is seen.
Magnetic field?

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S} - \boldsymbol{m} \cdot \boldsymbol{B}$$

Sun-Ko PRC2017; Wei-Deng-XGH PRC2019; Xie-Wang-Csernai PRC2017; Karpenko-Becattini EPJC2016

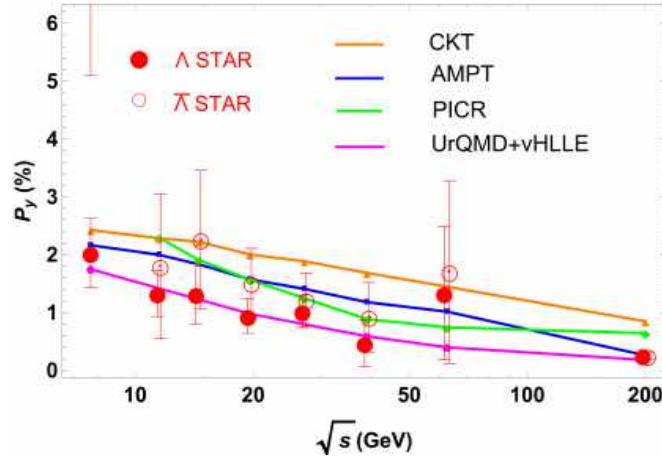
(Many similar results in literature)

Vorticity interpretation of global Λ polarization works well!

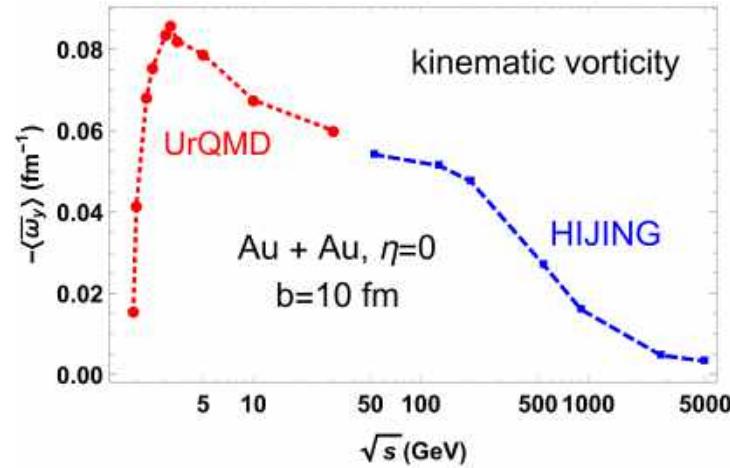
Global Λ spin polarization

The global polarization: Experiment = Theory

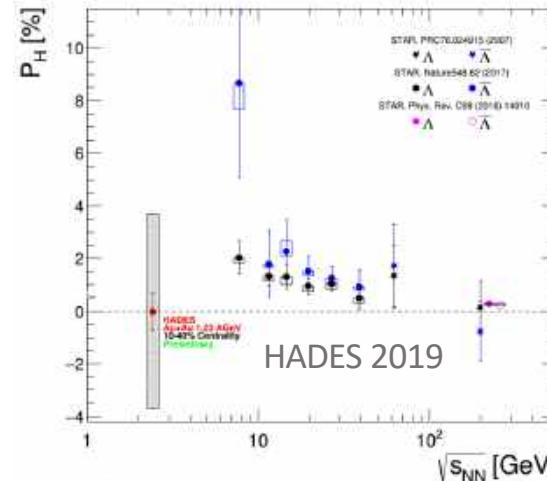
(Deng-XGH 2016; Deng-XGH-Ma-Zhang 2020)



VS

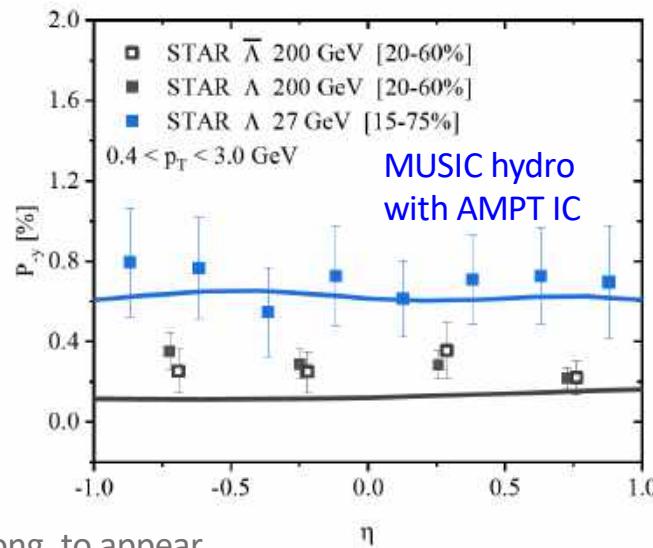
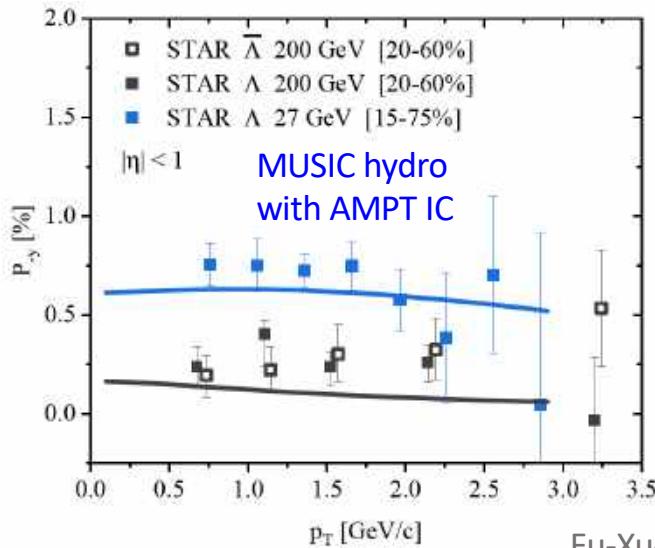


Need to study polarization
at very low \sqrt{s} : NICA,
FAIR, HIAF, BES II@RHIC?



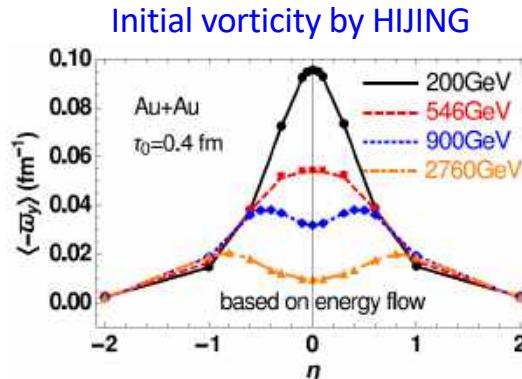
Differential Λ spin polarization

The global Λ polarization reflects the total amount of angular momentum retained in the (-1,1) rapidity region. How is it distributed in e.g. p_T , η , and ϕ ?

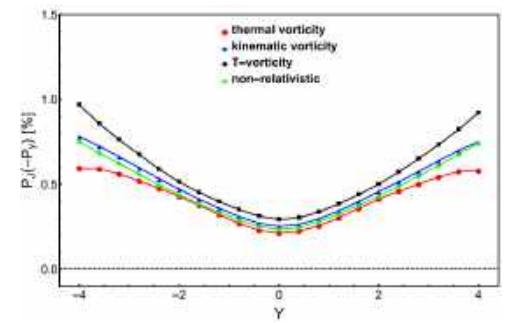


Fu-Xu-XGH-Song, to appear

Would be interesting to look at very large rapidity?



Deng-XGH PRC2016



Wu et al PRR2019

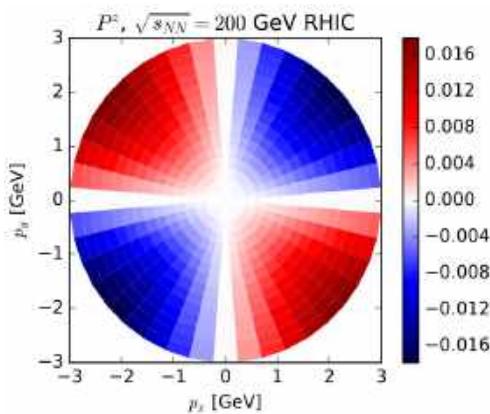
Differential Λ spin polarization

The global Λ polarization reflects the total amount of angular momentum retained in the (-1,1) rapidity region. How is it distributed in e.g. p_T , η , and ϕ ?

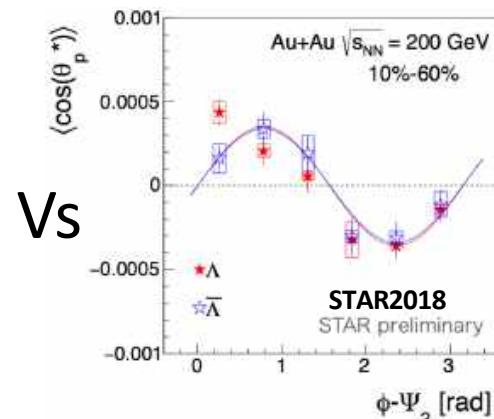
- Spin harmonic flow: $\frac{dP_{y,z}}{d\phi} \propto P_{y,z} + 2f_{2y,z}\sin(2\phi) + 2g_{2y,z}\cos(2\phi) + \dots$

1) longitudinal polarization vs ϕ

(Becattini-Karpenko PRL2018)



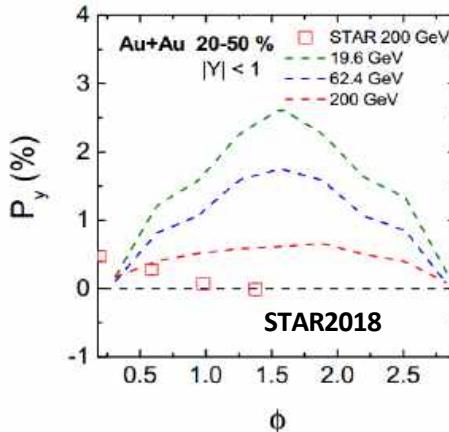
$$f_{2z}^{\text{ther}} < 0$$



$$f_{2z}^{\text{exp}} > 0$$

2) Transverse polarization vs ϕ

(Wei-Deng-XGH PRC2019)



$$g_{2y}^{\text{ther}} < 0, g_{2y}^{\text{exp}} > 0$$

We have a spin “sign problem”!

Differential Λ spin polarization

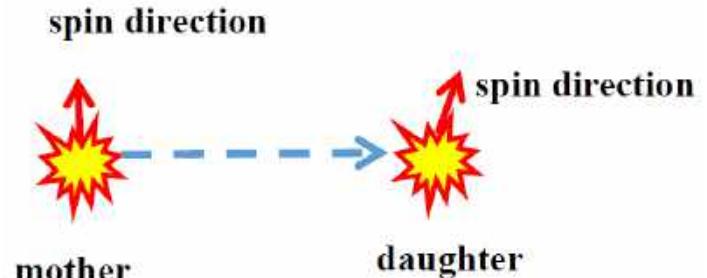
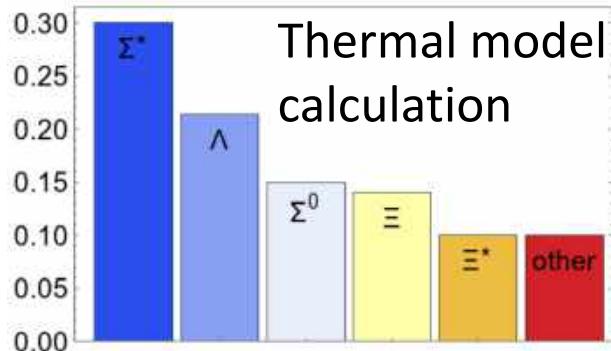
Attack the puzzles from theory side:

- Understand the vorticity (☺)
- Effect of feed-down decays (Xia-Li-XGH-Huang PRC2019, Becattini-Cao-Speranza EPJC2019)
(Measured Λ may come from decays of heavier particles)
- Go beyond equilibrium treatment (spin as a dynamic d.o.f)
spin hydrodynamics
spin kinetic theory
- Initial condition
(Initial polarization, initial flow,)
- Other possibilities
(chiral vortical effect (Liu-Sun-Ko 2019), mesonic mean-field (Csereai-Kapusta-Welle PRC2019), other spin chemical potential (Wu-Pang-XGH-Wang PRR2019, Florkowski et al 2019), contribution from gluons,)
- Other observables for vorticity and spin polarization
Vector meson spin alignment (Liang-Wang 2005; STAR and ALICE)
Vorticity dependent hadron yield (ExHIC-P Collaboration 2002.10082)

Feed-down effect

One important contribution

- About 80% of Λ 's are from decays of higher-lying particles



- Some decay channels can flip the spin, e.g., EM decay:

$$\Sigma^0 \rightarrow \Lambda + \gamma \quad \left(\frac{1}{2}\right)^+ \rightarrow \left(\frac{1}{2}\right)^+ 1^-$$

- The angular momentum conservation, requires that if Σ is polarization along the vorticity, its daughter Λ must be polarized opposite to the vorticity
- Let us examine the decay contribution

Spin transfer

- Consider the decay process

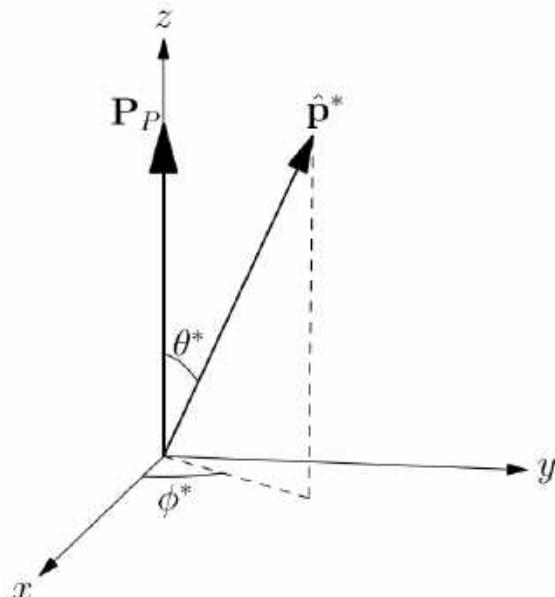


- The parent P is spin-polarized along z, the daughter D moves along \hat{p}^* in P's rest frame

Density matrix

$$\rho_{\lambda_D \lambda_X; \lambda'_D \lambda'_X}^f(\theta^*, \phi^*) = \sum_{M_P, M'_P} H_{\lambda_D \lambda_X; M_P} \rho_{M_P; M'_P}^i H_{M'_P; \lambda'_D \lambda'_X}^\dagger$$

$$|f\rangle = |\theta^* \phi^* \lambda_D \lambda_X\rangle \quad \longleftrightarrow \quad |i\rangle = |S_P M_P\rangle$$



The spin polarization of D:

$$\mathbf{P}_D = \text{tr}_D (\widehat{\mathbf{P}} \rho_{\lambda_D; \lambda'_D}^D) / \text{tr}_D (\rho_{\lambda_D; \lambda'_D}^D)$$

$$\rho_{\lambda_D; \lambda'_D}^D = \text{tr}_X (\rho_{\lambda_D \lambda_X; \lambda'_D \lambda'_X}^f)$$

Spin transfer

- For example, consider the EM decay $1/2^+ \rightarrow 1/2^+ 1^-$:

Initial density matrix:

$$\rho_{M_P; M'_P}^i = \text{diag} \left(\frac{1 + P_P}{2}, \frac{1 - P_P}{2} \right)$$

$$\xrightarrow{\hspace{1cm}} \rho_{\lambda_D \lambda_X; \lambda'_D \lambda'_X}^f = \frac{1}{8\pi} \begin{pmatrix} 1 + P_P \cos \theta^* & 0 & 0 & -P_P \sin \theta^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -P_P \sin \theta^* & 0 & 0 & 1 - P_P \cos \theta^* \end{pmatrix}$$

$$\xrightarrow{\hspace{1cm}} \rho_{\lambda_D; \lambda'_D}^D = \frac{1}{8\pi} \begin{pmatrix} 1 + P_P \cos \theta^* & 0 \\ 0 & 1 - P_P \cos \theta^* \end{pmatrix}$$

$$\xrightarrow{\hspace{1cm}} \mathbf{P}_D = -(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*) \hat{\mathbf{p}}^*$$

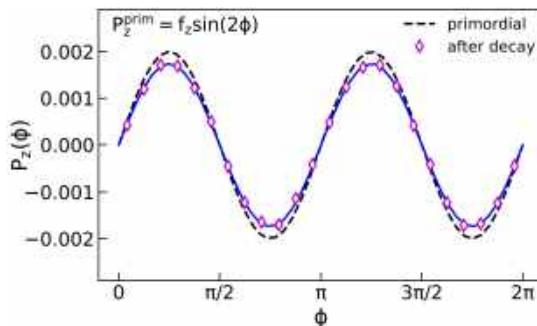
First derived by Gatto 1958

The feed-down correction

(Xia-Li-XGH-Huang PRC2019)

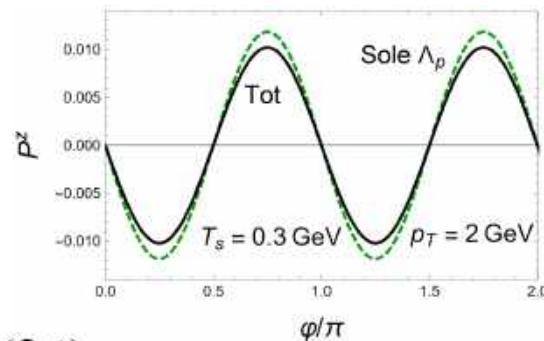
	spin and parity	$(1/N)dN/d\Omega^*$	\mathbf{P}_D	$\langle \mathbf{P}_D \rangle / \mathbf{P}_P$
strong decay	$1/2^+ \rightarrow 1/2^+ 0^-$	$1/(4\pi)$	$2(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*) \hat{\mathbf{p}}^* - \mathbf{P}_P$	-1/3
strong decay	$1/2^- \rightarrow 1/2^+ 0^-$	$1/(4\pi)$	\mathbf{P}_P	1
strong decay	$3/2^+ \rightarrow 1/2^+ 0^-$	$3[1 - 2\Delta/3 - (1 - 2\Delta) \cos^2 \theta^*]/(8\pi)$	Too long to be shown; see ref.	1
strong decay	$3/2^- \rightarrow 1/2^+ 0^-$	$3[1 - 2\Delta/3 - (1 - 2\Delta) \cos^2 \theta^*]/(8\pi)$		-3/5
weak decay	$1/2^- \rightarrow 1/2^- 0$	$(1 + \alpha P_P \cos \theta^*)/(4\pi)$		$(2\gamma + 1)/3$
EM decay	$1/2^+ \rightarrow 1/2^+ 1^-$	$1/(4\pi)$	$-(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*) \hat{\mathbf{p}}^*$	-1/3

- Longitudinal polarization
(Xia-Li-XGH-Huang PRC2019)



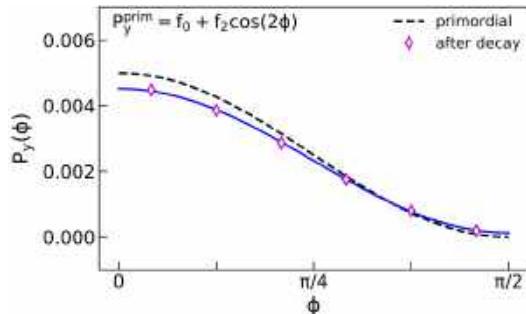
$$P_z = f_z \sin(2\phi)$$

(Becattini-Cao-Speranza EPJC2019)



- Transverse polarization

$$P_y = f_0 + f_2 \cos(2\phi)$$



Conclusion:

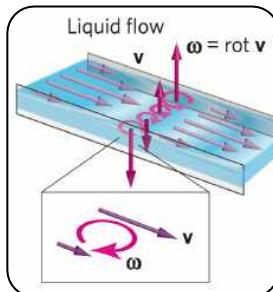
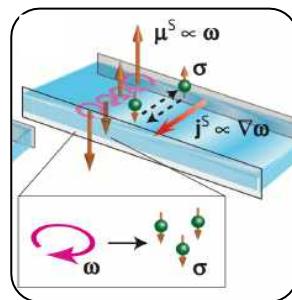
- Feed-down effects suppress $\sim 10\%$ Λ primordial spin polarization
- Do not solve the spin sign problem

Spin hydrodynamics

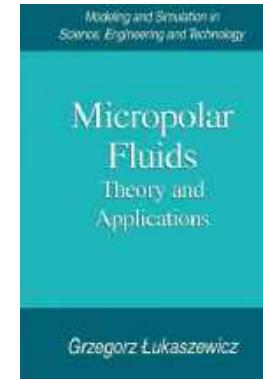
Spin hydrodynamics

Framework for collective spin dynamics. Spin as a (quasi-)hydrodynamic variable

- Widely used in non-relativistic spintronics, micropolar fluid,



(Takahashi et al Nat.Phys.2016)



- Relativistic ideal spin hydrodynamics (Florkowski et al PRC2018)

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$$

$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

$$S^{\lambda,\mu\nu} = \frac{w u^\lambda}{4\zeta} \omega^{\mu\nu}$$

(See also: Florkowski et al PRD2018, PPNP2019; Motenegro et al PRD2017, PRD2017)

Spin hydrodynamics

Relativistic **dissipative** spin hydrodynamics

(Hattori-Hongo-XGH-Matsuо-Taya PLB2019)

- Identify (quasi-)hydrodynamic variables: T and u^μ (4 for translation), $\omega^{\mu\nu} = -\omega^{\nu\mu}$ (spin chemical potential, 3 for rotation, 3 for boost).
- Derivative expansion. Apply 2nd law of thermodynamics.
- Constitutive relations up to $O(\partial)$

$$T_{(0)}^{\mu\nu} = eu^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu)$$

heat current

shear viscosity

bulk viscosity

$$T_{(1)}^{\mu\nu} = -2\kappa(Du^\mu + \beta\partial_\perp^\mu\beta^{-1})u^\nu - 2\eta\partial_\perp^{<\mu}u^{>\nu} - \zeta(\partial_\mu u^\mu)\Delta^{\mu\nu}$$

$$-2\lambda(-Du^\mu + \beta\partial_\perp^{[\mu}\beta^{-1} + 4u_\rho\omega^{\rho[\mu}})u^{\nu]}$$

boost heat current

rotational viscosity

- Hydrodynamic equations

$$\partial_\mu(T_{(0)}^{\mu\nu} + T_{(1)}^{\mu\nu} + O(\partial^2)) = 0$$

Energy-momentum conservation

$$\partial_\mu(u^\mu s^{\alpha\beta}) = T_{(1)}^{\beta\alpha} - T_{(1)}^{\alpha\beta} + O(\partial^2)$$

Angular momentum conservation

$$p = p(e, s^{\alpha\beta})$$

Equation of state

- Israel-Stewart type theory

$$\tau_\eta(D\sigma_\eta^{\mu\nu})_\perp + \sigma_\eta^{\mu\nu} = 2\eta\partial_\perp^{<\mu}u^{>\nu},$$

$$\tau_\zeta(D\sigma_\zeta^{\mu\nu})_\perp + \sigma_\zeta^{\mu\nu} = \zeta\theta\Delta^{\mu\nu},$$

$$\tau_\lambda(Dq^\mu)_\perp + q^\mu = \lambda(Du^\mu + \beta\partial_\perp^\mu T - 4\Omega^{\mu\nu}u_\nu),$$

$$\tau_\gamma(D\phi^{\mu\nu})_\perp + \phi^{\mu\nu} = 2\gamma(\partial_\perp^{[\mu}u^{>\nu]} + 2\Omega_\perp^{\mu\nu}),$$

(See also: Florkowski et al 2020; Shi-Gale-Jeon 2020)

Spin hydrodynamics

- Possible consequences: (1) New collective modes

$$\omega = -2iD_s,$$

$$\omega = -2iD_b,$$

$$\omega = \begin{cases} -2iD_s - i\gamma' k_z^2 + \mathcal{O}(k_z^4), \\ -i\gamma_\perp k_z^2 + O(k_z^4), \end{cases},$$

$$\omega = \begin{cases} \pm c_s k_z - i\frac{\gamma_{||}}{2} k_z^2 + \mathcal{O}(k_z^3), \\ -2iD_b - 2ic_s^2 \lambda' k_z^2 + \mathcal{O}(k_z^4). \end{cases}$$

↔ Longitudinal spin damping

↔ Longitudinal boost damping

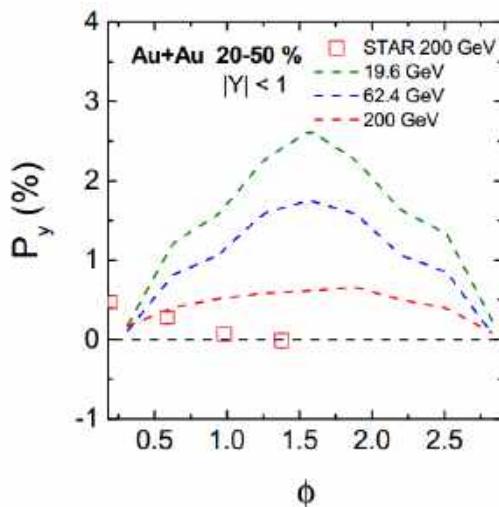
↔ Transverse spin damping

↔ Shear viscous damping

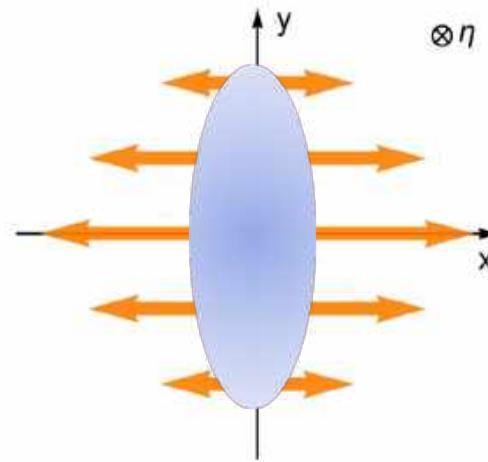
↔ Sound and bulk viscous damping

↔ Transverse boost damping

- (2) Polarization: azimuthal-dependence puzzle



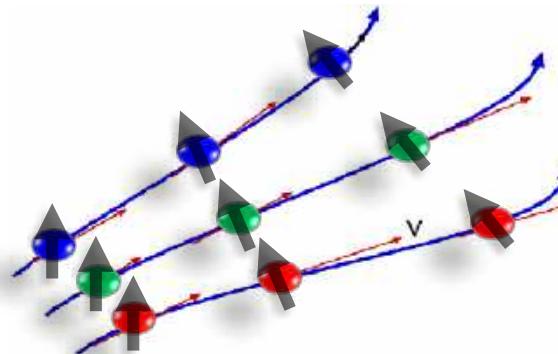
Spin elliptic flow?



Spin hydrodynamics

Future:

- Causal and stable (Israel-Stewart) 2nd order spin hydrodynamics
- Flow frame choice and pseudo-gauge choice, especially for Belinfante gauge
- Calculation of rotational viscosity and boost heat conductivity (insight to QCD)
- Formulate spin hydrodynamics for large vorticity at $O(1)$ and with magnetic field
- Derive spin hydrodynamics from kinetic theory or holography
- Application: numerical spin hydrodynamics for Λ polarization



Other observables

ϕ -spin alignment

- Vorticity can also polarize spin of vector mesons, e.g. ϕ
- Consider recombination $q + \bar{q} \rightarrow \phi$, the density matrix of q:

$$\rho^q = \frac{1}{2} \begin{pmatrix} 1 + P_q & 0 \\ 0 & 1 - P_q \end{pmatrix}$$

- The density matrix of ϕ is obtained from $\rho^q \otimes \rho^{\bar{q}}$ in basis of $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$ - $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$

$$\rho^V = \begin{pmatrix} \frac{(1+P_q)(1+P_{\bar{q}})}{3+P_q P_{\bar{q}}} & 0 & 0 \\ 0 & \frac{1-P_q P_{\bar{q}}}{3+P_q P_{\bar{q}}} & 0 \\ 0 & 0 & \frac{(1-P_q)(1-P_{\bar{q}})}{3+P_q P_{\bar{q}}} \end{pmatrix}$$

- Suppose $P_q = P_{\bar{q}}$,

$$\rho_{00}^{\rho(\text{rec})} = \frac{1 - P_q^2}{3 + P_q^2}$$

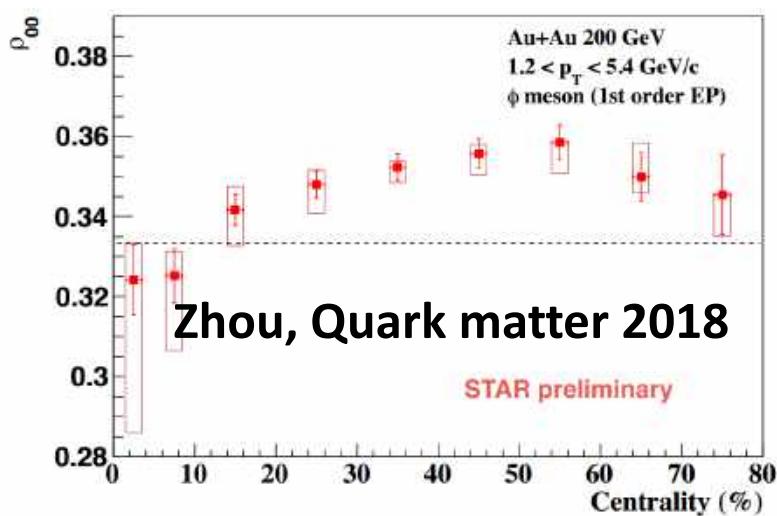
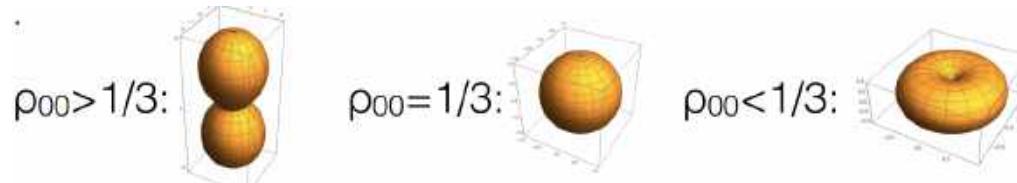
Liang-Wang 2005

Smaller than 1/3

ϕ -spin alignment

- Φ decay via strong process, no parity violation, it is not easy to determine its spin polarization states, but

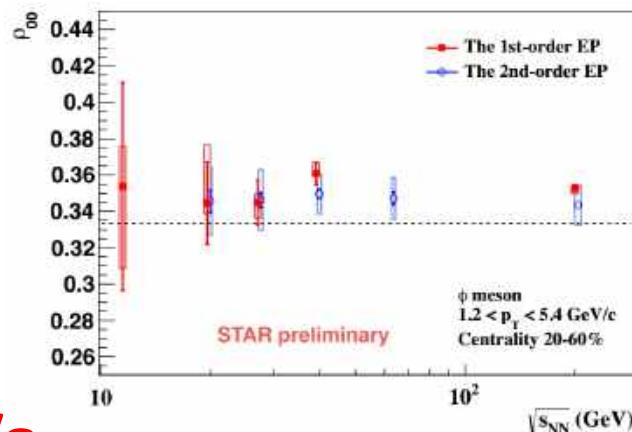
$$\frac{dN}{d(\cos \theta^*)} = N_0 \times [(1 - \rho_{00}) + (3\rho_{00} - 1)\cos^2 \theta^*]$$



Puzzle: for most centrality, ρ_{00} is far above 1/3?
Magnetic field contribution?
Mesonic field (Sheng-Oliva-Wang 2019)?
Gluon contribution?
... ...

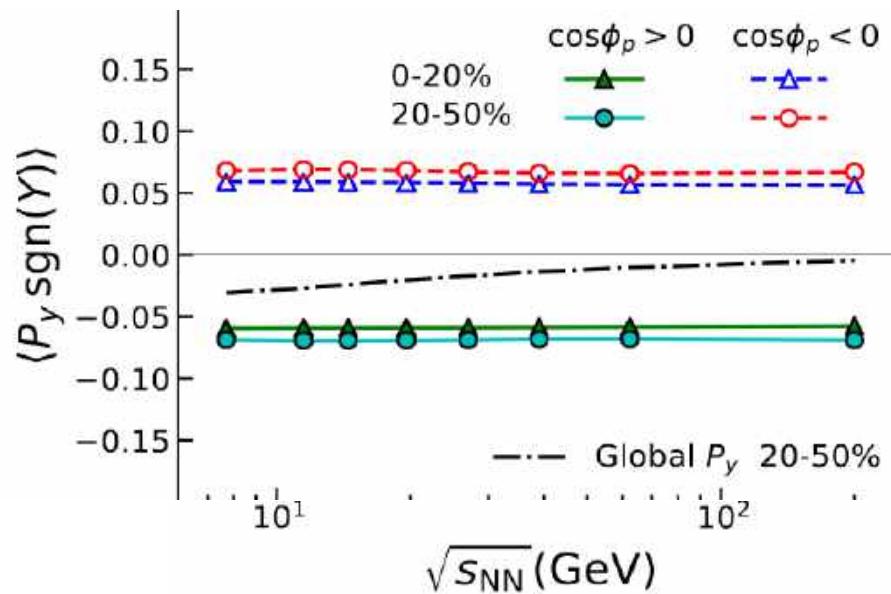
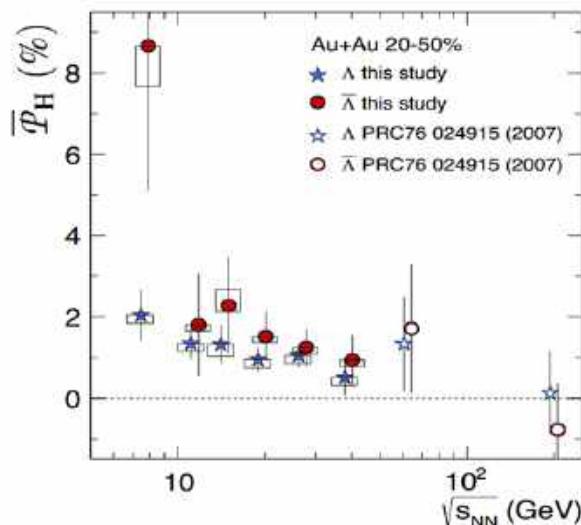
ϕ -spin alignment

- Φ decay via strong process, no parity violation, it is not easy to determine its spin polarization states, but



No significant energy dependence

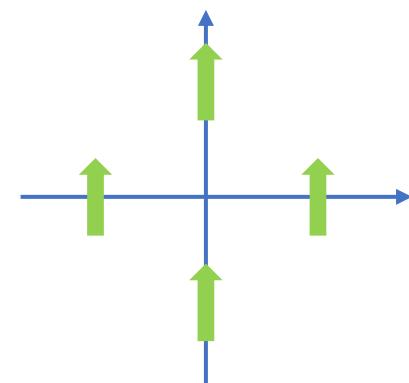
May be understood. As ρ_{00} depends on P_q^2



ϕ -spin alignment

- Spin configuration for vector mesons:

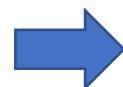
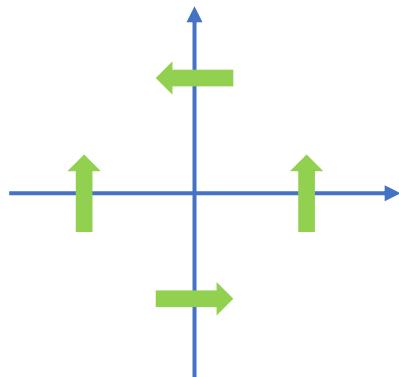
$$\rho_{11} \sim |\uparrow\uparrow\rangle (\uparrow\uparrow|, \rho_{-1-1} \sim |\downarrow\downarrow\rangle (\downarrow\downarrow|, \rho_{00} \sim [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle] [(\uparrow\downarrow| - (\downarrow\uparrow|)]$$



$$\rho_{00} = \frac{1 - P_y^2}{3 + P_y^2}$$

Liang-Wang 2005

$$\rho_{00}^{V(rec)} < 1/3 \quad \text{for } q^\uparrow + \bar{q}^\uparrow \rightarrow V$$

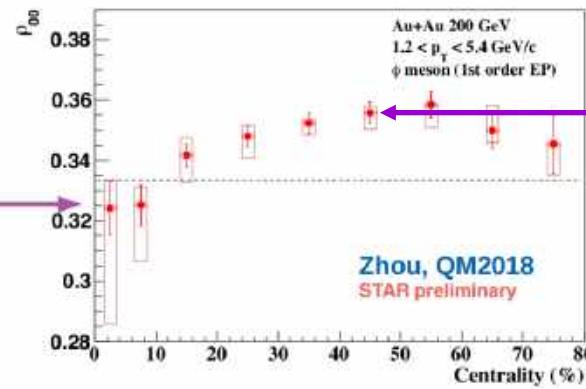
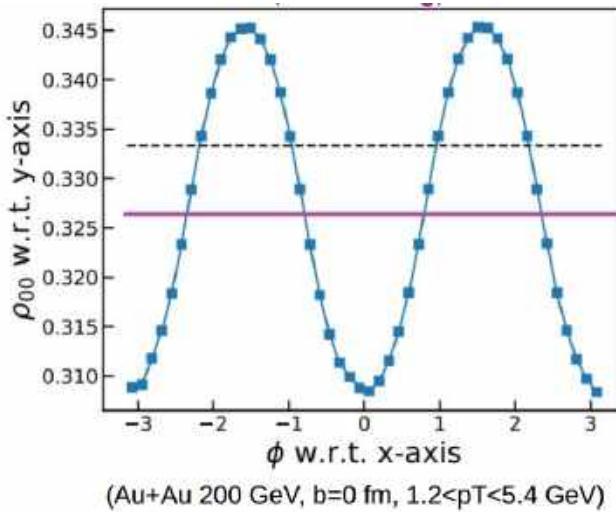


$$\rho_{00} = \frac{1 - P_y^2 + P_x^2 + P_z^2}{3 + P^2}$$

Xia-Li-XGH-Huang, to appear

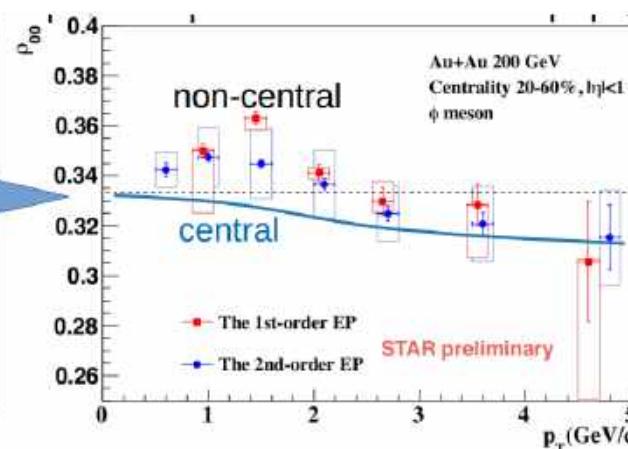
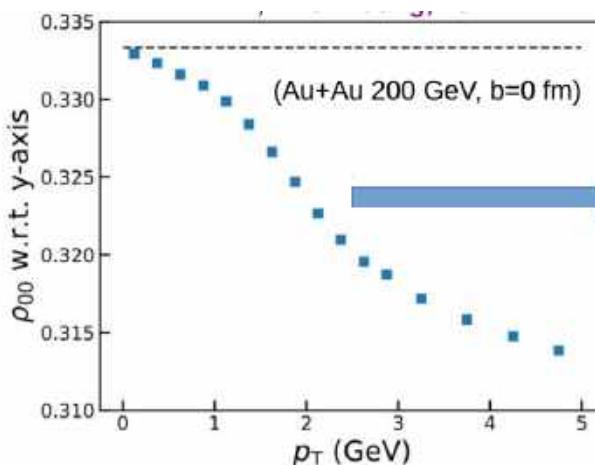
ϕ -spin alignment

- Predictions for central collisions:



Noncentral collisions:
Magnetic field ?

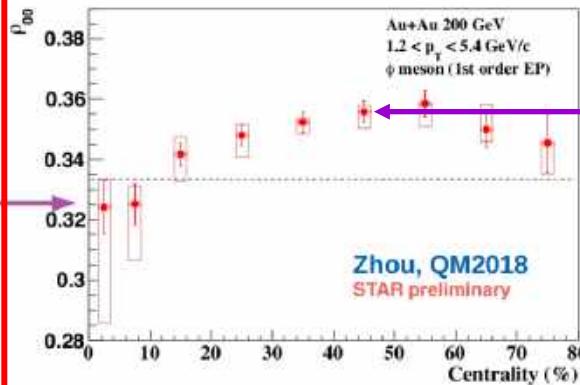
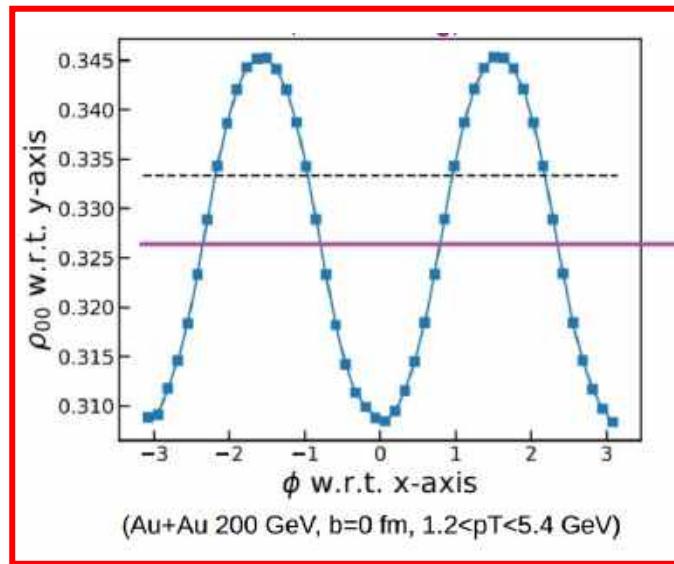
$$\rho_{00}^{mag} = \frac{1 + P_y^2}{3 - P_y^2} > \frac{1}{3}$$



$$\rho_{00}^{vor} = \frac{1 - P_y^2 + P_x^2 + P_z^2}{3 + P^2}$$

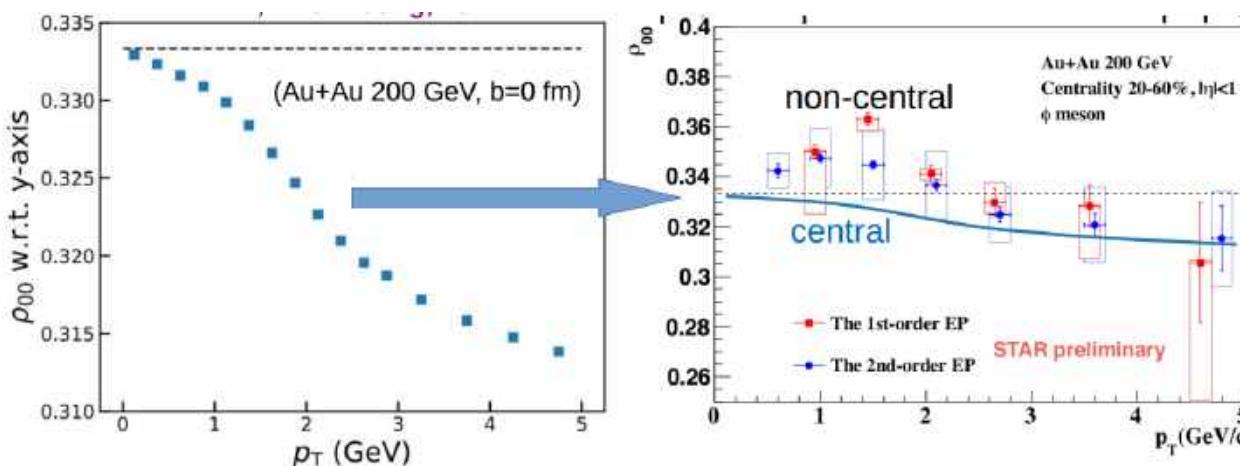
ϕ -spin alignment

- Predictions for central collisions:



Noncentral collisions:
Magnetic field ?

$$\rho_{00}^{mag} = \frac{1 + P_y^2}{3 - P_y^2} > \frac{1}{3}$$



Well testable!
Evidence of
circular vorticity

Spin dependent hadron yields

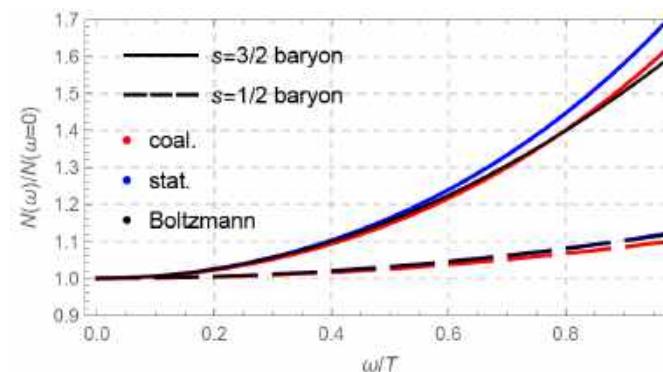
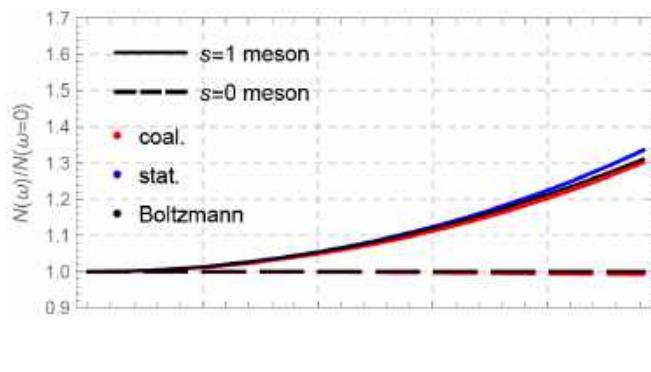
Vorticity is the “spin chemical potential” (ExHIC-P Collaboration 2002.10082)

$$E_h = \sqrt{m_h^2 + \mathbf{p}^2} - \boldsymbol{\mu}^{\text{ch}} \cdot \mathbf{Q}_h - \omega^{\text{ch}} s_z$$



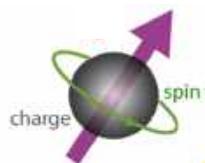
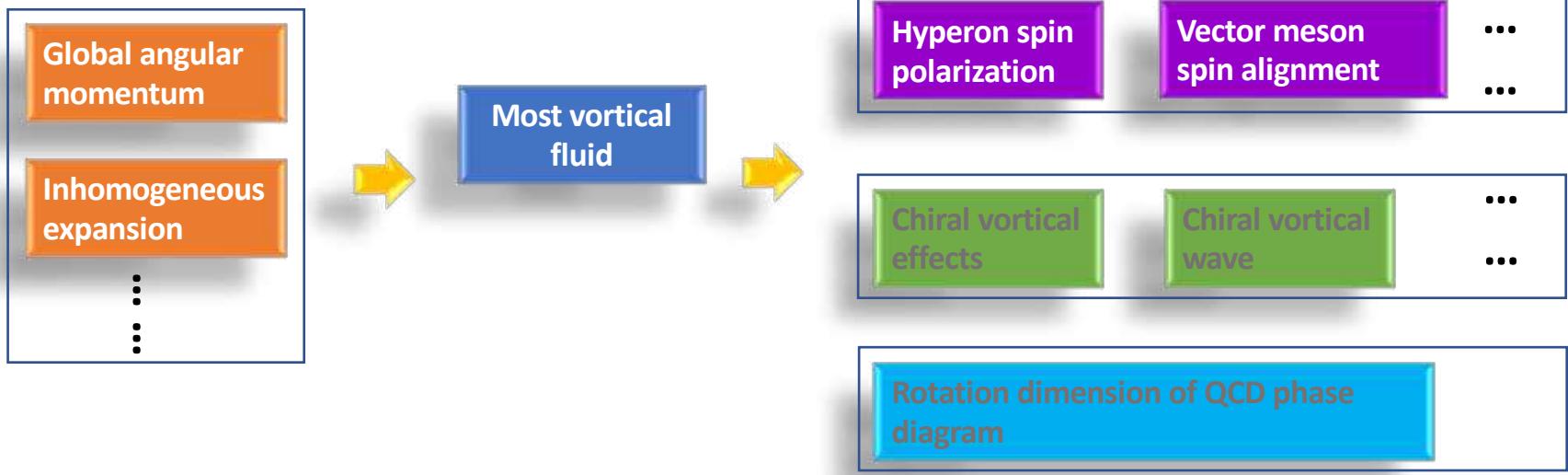
$$\frac{N^{\text{stat/coal}}(\omega)}{N^{\text{stat/coal}}(\omega = 0)} \sim 1 + \frac{s(1+s)}{6} \left(\frac{\omega}{T} \right)^2$$

Naively, it is the same order as ρ_{00} , could be cross-check of vector spin alignment



Observable: ratio of e.g. $\frac{N_\phi}{N_K}$ or $\frac{N_\Omega}{N_\Xi}$ as function of centrality and energy

Summary

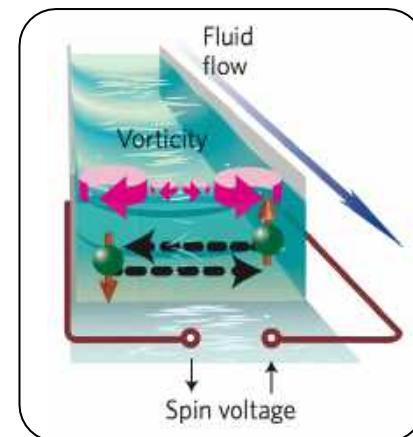
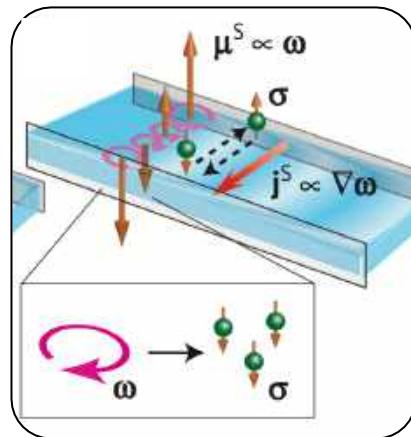
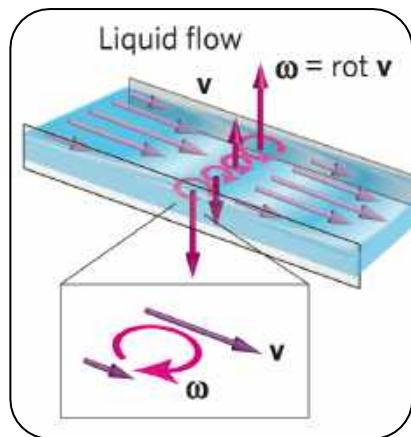


Heavy-ion physics: electronics era to spintronics era
Puzzles, challenges, but opportunities

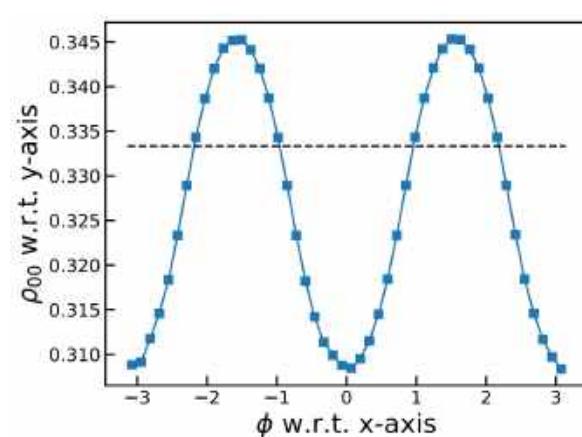
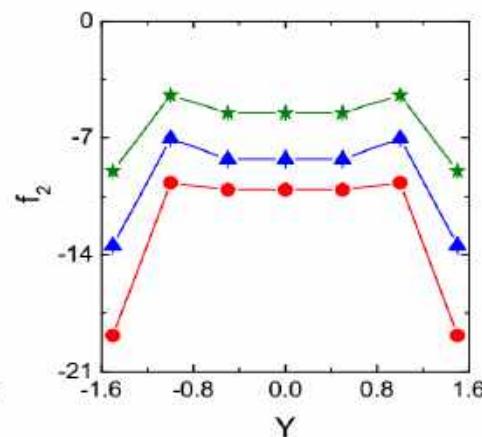
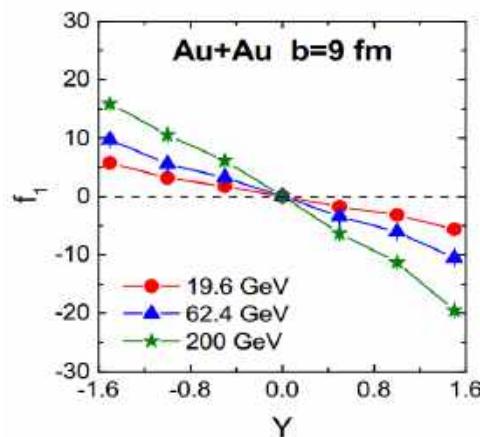
Thank you

Subatomic spintronics

- Spin hydrodynamic generation in Hg (Takahashi, et al. Nat. Phys. (2016))



- Subatomic spintronics in HIC: a new probe for QGP



Angular momentum in HIC

