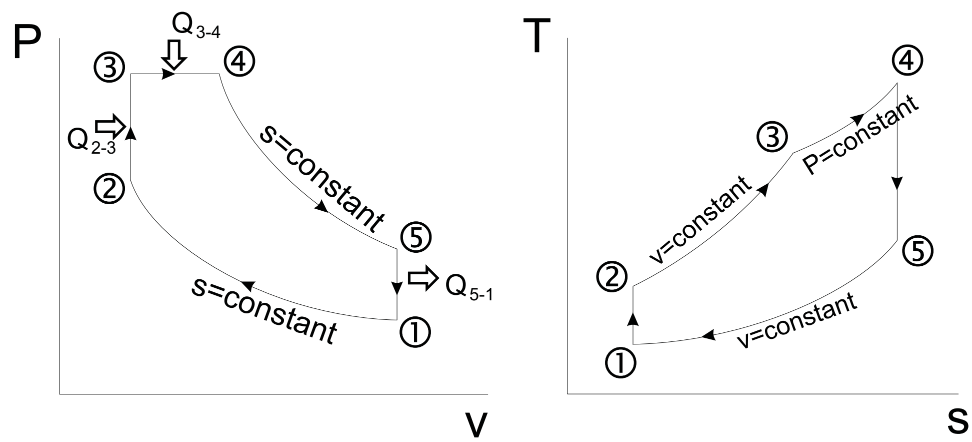
سوال 1

Part a)



State	$P\ (kPa)$	$T\ (K)$
1	100	293.15
2	2511.9	735.98
3	10434.36	3057.24
4	10434.36	3886.43
5	1326.44	1703.10

Process 1 - 2

The process is isentropic so

$$Pv^k = const$$

where $k = 1.4$

We know from the isentropic equations

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

and

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k = (10)^{1.4} = 25.119$$

Then

$$P_2 = (100\ kPa)(25.119) = 2511.9\ kPa \Leftarrow$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (20 + 273)(10)^{0.4} = 735.98\ K \Leftarrow$$

Process 2 - 3
the volume is constant during this process. Performing a 1st law balance gives

$$q_{2-3} = c_v(T_3 - T_2)$$

At 300 K we know that $c_v = 0.718\ kJ/kg \cdot K$ for air. The problem states that **2/3** of the total heat input occurs during this phase of the cycle.

$$\left(\frac{2}{3}\right) 2500\ kJ/kg = (0.718\ kJ/kg \cdot K)(T_3 - 735.98)$$

$$T_3 = 3057.24\ K \Leftarrow$$

For an ideal gas (with constant mass) we know

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}$$

But $V_2 = V_3$, so

$$\frac{P_3}{T_3} = \frac{P_2}{T_2}$$

$$P_3 = T_3\ \frac{P_2}{T_2} = (3057.24\ K)\left(\frac{2511.9\ kPa}{735.98\ K}\right) = 10434.36\ kPa \Leftarrow$$

Process 3 - 4
constant pressure process. We know for this case that

$$q_{3-4} = c_p(T_4 - T_3)$$

Therefore

$$\left(\frac{1}{3}\right) 2500\ kJ/kg = (1.005\ kJ/kg \cdot K)(T_4 - 3057.24)$$

$$T_4 = 3886.43\ K \Leftarrow$$

We know

$$P_4 = P_3 = 10434.36\ kPa \Leftarrow$$

Process 4 - 5

This is an isentropic process where

$$\frac{P_5}{P_4} = \left(\frac{V_4}{V_5}\right)^k$$

and it can be shown that

$$\frac{P_5}{P_4} = \left(\frac{V_2}{V_1}\,\frac{T_4}{T_3}\right)^k$$

or

$$P_5 = (10434.36\ kPa)\left(\frac{1}{10}\,\frac{3886.43\ K}{3057.24\ K}\right) = 1326.44\ kPa$$

and

$$T_5 = T_4\left(\frac{P_5}{P_4}\right)^{(k-1)/k} = 3886.43\left(\frac{581.27}{10434.36}\right)^{0.4/1.4} = 1703.10\ K$$

Part b)

The thermal efficiency is given as

$$\begin{aligned} \eta &= \frac{\text{net work done}}{\text{total heat in}} \\ &= \frac{w_{1-2} + w_{3-4} + w_{4-5}}{q_{in}} \end{aligned}$$

$$\begin{aligned} w_{1-2} &= -c_v(T_2 - T_1) = -(0.718\ kJ/kg \cdot K)(735.98\ K - 293\ K) = -318.06\ kJ/kg \\ w_{3-4} &= q_{3-4} - c_v(T_4 - T_3) \\ &= \left(\frac{1}{3}\right)(2500\ kJ/kg) - (0.718\ kJ/kg \cdot K)(3886.43\ K - 3057.24\ K) \\ &= 237.97\ kJ/kg \\ w_{4-5} &= -c_v(T_5 - T_4) = -(0.718\ kJ/kg \cdot K)(1703.1\ K - 3886.43\ K) = 1567.63\ kJ/kg \end{aligned}$$

Therefore the efficiency is

$$\eta = \frac{-318.06 + 237.97 + 1567.63}{2500} = 59.5\%$$

Part c)

We know that $\boldsymbol{V_2 = V_3}$ and $\boldsymbol{V_1 = V_5}$ and for the isentropic process between $\mathbf{4-5}$

$$\frac{V_4}{V_5} = \frac{V_4}{V_1} = \left(\frac{P_5}{P_4}\right)^{1/k}$$

and

$$r = \frac{V_1}{V_2} = \frac{V_1}{V_3}$$

Therefore the cutoff ratio is

$$r_v = \frac{V_4}{V_3} = \frac{V_1(P_5/P_4)^{1/k}}{V_1/r} = (P_5/P_4)^{1/k} \cdot r = 10 \cdot \left(\frac{581.27}{10434.36}\right)^{1/1.4} = 1.27$$

$$r_p = \frac{P_3}{P_2} = \frac{10434.36}{2511.9} = 4.154$$

$$\begin{aligned}
\eta &= 1 - \frac{r_p r_v^k - 1}{[(r_p - 1) + k r_p (r_v - 1)] r^{k-1}} \\
&= 1 - \frac{(4.154)(1.27)^{1.4} - 1}{[(4.154 - 1) + (1.4)(4.154)(1.27 - 1)](10)^{0.4}} \\
&= 0.595 \Leftarrow
\end{aligned}$$

Using the isentropic relationships, T_{2s} can be calculated as

$$T_{2s} = T_1 \times (r_p)^{k-1/k} = (25 + 273) \times (18)^{0.4/1.4} = 680.56\ K$$

Since the compressor efficiency is 100%, T_2 is the same as T_{2s}

$$T_2 = T_{2s} = 680.56\ K$$

Using the isentropic relationship for the turbine

$$T_{4s} = T_3 \times \left(\frac{1}{r_p}\right)^{k-1/k} = (1000 + 273) \times \left(\frac{1}{18}\right)^{0.4/1.4} = 557.42\ K$$

Since the mass flow rate and the specific heat are constant, the isentropic efficiency of the turbine can be written as

$$\begin{aligned} \eta_T &= \frac{T_3 - T_4}{T_3 - T_{4s}} \\ 0.85 &= \frac{1273 - T_4}{1273 - 557.42} \end{aligned}$$

Therefore

$$T_4 = 1273 - 0.85(1273 - 557.42) = 664.76\ K$$

The net work output per unit mass is

$$\begin{aligned} \frac{W_{net}}{m} &= c_p(T_3 - T_4) - c_p(T_2 - T_1) \\ &= 1.004\ kJ/kg \cdot K\ [(1273 - 664.76) - (680.56 - 298)]\ K \\ &= 226.58\ kJ/kg \end{aligned}$$

Part a)

The net power output is

$$Power = \dot{m} \times \frac{W_{net}}{m} = 10\ kg/s\ \times\ 226.58\ kJ/kg = 2265.8\ kW \Leftarrow$$

Part b)

The thermal efficiency of the engine is

$$\begin{aligned} \eta &= \frac{W_{net}}{Q_{in}} = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_2} \\ &= \frac{1273 - 664.76 - 680.56 + 298}{1273 - 680.56} \\ &= 0.381 = 38.1\% \Leftarrow \end{aligned}$$

Note: $\eta = 1 - (r_p)^{(1-k)/k}$ can only be used for the ideal Brayton cycle where $T_4 = T_{4s}$